

Reliability Evaluation of Integrated Modular Avionics Based on Multi-State System with Performance Sharing Mechanism

Gengshuo Hu
School of Reliability and Systems Engineering,
Beihang University
Beijing, China
jiaojian@buaa.edu.cn

Jian Jiao*
School of Reliability and Systems Engineering,
Beihang University
Beijing, China
jiaojian@buaa.edu.cn

Langxiao Hu
School of Reliability and Systems Engineering,
Beihang University
Beijing, China
jiaojian@buaa.edu.cn

Yongfeng Jing
School of Reliability and Systems Engineering,
Beihang University
Beijing, China
jiaojian@buaa.edu.cn

Abstract—To solve the problem of reliability modeling and evaluation of integrated modular avionics (IMA), this study extends the multi-state system with common bus performance sharing based on the stricture and work principles of IMA. Subsequently, based on the extended model, the universal generating function is applied to evaluate the reliability of IMA. Finally, a case study on the integrated task processing system in IMA is used to analyze the impact of changes in the number of spare general processing units on system reliability, with the aim is to validate the effectiveness and practicality of the proposed model.

Keywords—integrated modular avionics, multi-state model, universal generating function, reliability

I. INTRODUCTION

The avionics system is the core system of an aircraft, providing essential functions such as human-machine interaction and navigation. The cost of avionics systems accounts for an increasing proportion of aircraft design and manufacturing costs [1]. Integrated modular avionics (IMA) has gradually become a mainstream approach owing to its advantages in terms of cost reduction and performance improvement. It has been widely applied in aircrafts such as the Boeing 787 and Airbus A380 [2]. Meanwhile, owing to its extensive applications, the demand for highly reliable IMA has been increasing in the aviation industry [3]. Therefore, research on ensuring the high reliability of IMA has become essential [4].

Many researchers have proposed several reliability modeling and assessment methods for IMA, such as the method of combining fault trees [5], Petri nets, and the finite state machine method [6]. However, the quantitative modeling assessment of IMA reliability is still in its nascent stage. Meanwhile, the surplus data processing ability of some general processing units (GPUs) can be transferred to other GPUs with insufficient computational resources through a communication system [7]. Therefore, this study draws on a multi-state system (MSS) with a common bus to propose a quantitative reliability modeling and assessment method for IMA.

Multi-state systems that consider a performance-sharing mechanism are widely used in industry and academia [8]. Some engineering systems such as heating systems [9] and

distributed computer systems [10] can be modeled using the performance-sharing mechanism of an MSS. In recent years, many researchers have studied extended models that consider performance-sharing mechanisms [11-15]. These studies cover a wide range of aspects, such as instantaneous availability analysis of an MSS [11] and optimal allocation and maintenance problems [12].

However, certain extended models have not been discussed in previous studies, such as the extended model, which can be used to assess IMA reliability. The data computation and processing ability of IMA are mainly realized using a GPU. The GPU is composed of a multicore processor, memory, and transmission links, where the remaining computational resources of the processor and memory in the GPU can be transferred to another processor and memory with insufficient computational resources using the common bus among them. The reliability of the system is underestimated without considering the performance-sharing mechanism, while its reliability is overestimated if the performance sharing of multiple buses is not considered. Therefore, in this study, we extended the model to quantitatively assess IMA reliability.

II. MODEL ILLUSTRATION

The system consists of m independent subsystems connected through two common buses, and its structure is shown in Fig. 1. Each subsystem consists of two different types of components: components 1, 3, ..., $2m-1$ are non-identical multi-state weighted k -out-of- n systems [16] and are connected by common bus 1. Components 2, 4, ..., $2m$ are connected by common bus 2. Each component is composed of one or several units. Each unit has several states: the random performance $G_{i,j}$ of unit j in component i takes numeric values from the set $g_{i,j,h} = \{g_{i,j,1}, g_{i,j,2}, \dots, g_{i,j,H_{i,j}}\}$, and the random demand $W_{i,j}$ takes numeric values from the set $w_{i,j,p_i} = \{w_{i,j,1}, w_{i,j,2}, \dots, w_{i,j,P_i}\}$.

For components 1, 3, ..., $2m-1$, the surplus performance of the components must satisfy their thresholds. For components 2, 4, ..., $2m$, the demand must satisfy the performance. If the redistributable surplus performance in a component remains, it

can be transferred to other components with performance deficiencies by a common bus. However, the transmission capacities of the common buses are limited. For components connected by common bus 1, when the surplus performance exceeds the threshold [16], the component can be in the

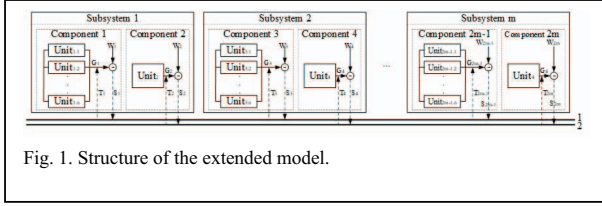


Fig. 1. Structure of the extended model.

working state. For the components connected by common bus 2, when the performance of the component satisfies its demand, the component can operate. When the two components in the subsystem operate, the subsystem can function regularly. The system operates when the number of working subsystems satisfies a specified quantity.

III. MODEL ANALYSIS

According to the extended model in the previous section, the performance surplus S_i and performance deficiency D_i of each component can be written as (1) and (2), respectively:

$$S_i = \max(G_i - W_i, 0), \quad (1)$$

$$D_i = \max(W_i - G_i, 0), \quad (2)$$

where G_i and W_i are the random performance and random demand of component 1, respectively.

For common bus 1, which consists of components 1, 3, ..., $2m-1$, the performance surplus and deficiency vectors can be expressed as (3) and (4), respectively.

$$S_{tub_{e1}} = \{S_1, S_3, \dots, S_{2m-1}\} \quad (3)$$

$$D_{tub_{e1}} = \{D_1, D_3, \dots, D_{2m-1}\} \quad (4)$$

For common bus 2, which consists of components 2, 4, ..., $2m$, the performance surplus and deficiency vectors can be expressed as (5) and (6), respectively [9].

$$S_{tub_{e2}} = \sum_{tub_{e2}} S_i \quad (5)$$

$$D_{tub_{e2}} = \{D_2, D_4, \dots, D_{2m}\} \quad (6)$$

On account of the performance-sharing mechanism, the redistributable performance surplus of common bus 1 refers to the portion of the performance surplus of each component that exceeds the system-defined threshold while considering the transmission capacity of bus 1. For bus 2, the redistributable performance surplus is the minimum value between the sum of the performance surplus and transmission capacity of each component.

$$T_{tub_{e1}} = \min\left(\sum_{tub_{e1}} \max((G_i - W_i - K_i), 0), C_1\right) \quad (7)$$

$$T_{tub_{e2}} = \min\left(\sum_{tub_{e2}} \max((G_i - W_i), 0), C_2\right) \quad (8)$$

Here, $T_{tub_{e1}}$ and $T_{tub_{e2}}$ represent the total number of remaining performances that can be redistributed to common bus 1 and bus 2, respectively; C_1 and C_2 are the transmission capacities of the common bus 1 and bus 2, respectively; K_i is the performance surplus threshold of the components 1, 3, ..., $2m-1$. For bus 1, the redistributable performance surplus is prioritized for transfer to components whose performance deficiency is minimal. When there are no components with performance deficiencies, the performance surplus is transferred to the components with a performance surplus closer to the threshold. For bus 2, the redistributable performance surplus is prioritized for transfer to the components with minimal performance deficiencies [9].

After performance sharing, the performance surplus $\bar{S}_{tub_{e1}}$ and performance deficiency $\bar{D}_{tub_{e2}}$ of the components connected by common bus 1 and bus 2 can be defined as:

$$\bar{S}_{tub_{e1}} = \{\bar{S}_1, \bar{S}_3, \dots, \bar{S}_{2m-1}\} \quad (9)$$

$$\bar{D}_{tub_{e2}} = \{\bar{D}_2, \bar{D}_4, \dots, \bar{D}_{2m}\} \quad (10)$$

The set of thresholds of components 1, 3, ..., $2m-1$ is given by:

$$K = \{K_1, K_3, \dots, K_{2m-1}\} \quad (11)$$

In Fig. 1, the subsystem i is composed of components i_{2m-1} and i_{2m} . When the performance surplus of component i_{2m-1} connected by bus 1 exceeds the specified threshold and no performance deficiencies exist in component i_{2m} connected by bus 2, subsystem i is considered to be in working state. In this paper, L represents the quantity of subsystems in the working state.

$$L = \sum_{i=1}^m I(\bar{S}_{tub_{e1}}(i) \geq K(i), \bar{D}_{tub_{e2}}(i) = 0) \quad (12)$$

Here, $I(b)$ is the indicative function of event b .

The reliability of the system is derived as the probability that the number of subsystems in the working state exceeds the system-specified quantity n .

$$R = \Pr\{L \geq n\} \quad (13)$$

IV. SYSTEM RELIABILITY ASSESSMENT

Owing to the effectiveness of the universal generating function (UGF) as a method for describing an MSS and its widespread application in reliability assessments [10], this study uses this technology to assess the system reliability.

1) The probability mass function (PMF) of the $Unit_{i,j}$ performance in the UGF is defined as:

$$u_{i,j}(z) = \sum_{h_{i,j}=1}^{H_{i,j}} a_{i,j,h_{i,j}} z^{g_{i,j,h_{i,j}}}, \quad (14)$$

where $a_{i,j,h_{i,j}} = \Pr\{G_{i,j} = g_{i,j,h_{i,j}}\}$.

2) The PMF of the Component_i demand in the UGF is defined as:

$$\omega_i(z) = \sum_{p_i=1}^{P_i} \beta_{i,p_i} z^{w_{i,p_i}}, \quad (15)$$

where $\beta_{i,p_i} = \Pr\{W_i = w_{i,p_i}\}$.

3) According to a previous study [16], the performance G_i of components 1, 3, ..., $2m-1$ refers to the performance summation of the units in the component. $\oplus$ denotes the iterative calculation of the performance of components 1, 3, ..., $2m-1$. The PMF of components 1, 3, ..., $2m-1$ in the UGF can be obtained as:

$$u_i(z) = u_{i,1}(z) \oplus \dots \oplus u_{i,j}(z) \quad (16)$$

$$= \sum_{h_i=1}^{H_i} a_{i,h_i} z^{g_{i,h_i}},$$

where $a_{i,h_i} = \Pr\{G_i = g_{i,h_i}\}$.

4) Using composition operator $\otimes$ to obtain the joint UGF presentation of the S_i and D_i of component i , we define $\Delta_i(z)$ as:

$$\Delta_i(z) = u_i(z) \otimes \omega_i(z) \quad (17)$$

$$= \sum_{h_i=1}^{H_i} \sum_{p_i=1}^{P_i} a_{i,h_i} \beta_{i,p_i} z^{\max(0, g_{i,h_i} - \omega_{i,p_i}), \max(0, \omega_{i,p_i} - g_{i,h_i})}$$

$$= \sum_{v_i=1}^{V_i} \gamma_{i,v_i} z^{s_{i,v_i}, d_{i,v_i}},$$

where V_i represents the number of terms after combining similar terms, s_{i,v_i} is the remaining performance of component i , d_{i,v_i} is the performance deficiency of component i , and $\gamma_{i,v_i} = \Pr\{S_i = s_{i,v_i}, D_i = d_{i,v_i}\}$ is the probability of the corresponding joint event.

5) Operator $\oplus$ is applied iteratively over the component connected by common bus 1. The UGF for the performance surplus and deficiency vectors S_{tube_1} and D_{tube_1} can be described as:

$$U_{\Omega_1}(z) = \Delta_1(z) \oplus \Delta_3(z) \oplus \dots \oplus \Delta_{2m-1}(z) \quad (18)$$

$$= \sum_{v_1=1}^{V_1} \sum_{v_3=1}^{V_3} \dots \sum_{v_{2m-1}=1}^{V_{2m-1}} \left(\prod_{i=1}^{\frac{m}{2}} \gamma_{2i-1, v_{2i-1}} \right) z^{\{s_{1,v_1}, s_{2,v_2}, \dots, s_{2m-1, v_{2m-1}}\}, \{d_{1,v_1}, d_{2,v_2}, \dots, d_{2m-1, v_{2m-1}}\}}$$

$$= \sum_{f_{tube_1}=1}^{F_{tube_1}} \gamma_{f_{tube_1}} z^{s_{f_{tube_1}}, d_{f_{tube_1}}},$$

where F_{tube_1} represents the number of terms after combining similar terms, $s_{f_{tube_1}}$ and $d_{f_{tube_1}}$ denote the performance surplus and performance deficiency of the components in common bus 1, respectively, and $\gamma_{f_{tube_1}} = \Pr\{S_{tube_1} = s_{f_{tube_1}}, D_{tube_1} = d_{f_{tube_1}}\}$ is the probability of the corresponding joint event.

6) Operator $\oplus$ is applied iteratively over the component connected by common bus 2, and the UGF for the performance surplus vector S_{tube_2} and performance deficiency D_{tube_2} can be described as:

$$U_{\Omega_2}(z) = \Delta_2(z) \oplus \Delta_4(z) \oplus \dots \oplus \Delta_{2m}(z) \quad (19)$$

$$= \sum_{v_2=1}^{V_2} \sum_{v_4=1}^{V_4} \dots \sum_{v_{2m}=1}^{V_{2m}} \left(\prod_{i=1}^{\frac{m}{2}} \gamma_{2i, v_{2i}} \right) z^{\{s_{2,v_2}, \{d_{2,v_2}, d_{4,v_4}, \dots, d_{2m, v_{2m}}\}\}}$$

$$= \sum_{f_{tube_2}=1}^{F_{tube_2}} \gamma_{f_{tube_2}} z^{s_{f_{tube_2}}, d_{f_{tube_2}}},$$

where F_{tube_2} represents the number of terms after combining similar terms, $s_{f_{tube_2}}$ and $d_{f_{tube_2}}$ denote the performance surplus and performance deficiency of the components in common bus 2, respectively, and $\gamma_{f_{tube_2}} = \Pr\{S_{tube_2} = s_{f_{tube_2}}, D_{tube_2} = d_{f_{tube_2}}\}$ is the probability corresponding to the joint event.

7) For bus 1 and bus 2, the random transmission capacity set can be expressed as $c_{q,j_q} = \{c_{q,1}, c_{q,2}, \dots, c_{q,J_q}\}$. The transmission capacity for bus 1 and bus 2 in the UGF can be described as:

$$\eta_q(z) = \sum_{j_q=1}^{J_q} \delta_{q,j_q} z^{c_{q,j_q}}, \quad (20)$$

where $\delta_{q,j_q} = \Pr\{C_q = c_{q,j_q}\}$ is the probability density function of the transmission capacity of the common bus.

8) Under the sharing mechanism, the UGF of $\bar{S}_{tube_1}$ and $\bar{D}_{tube_1}$ can be calculated using operator $\odot$ as follows:

$$\bar{U}_{\Omega_1}(z) = U_{\Omega_1}(z) \odot \eta_1(z) \quad (21)$$

$$= \sum_{f_{tube_1}=1}^{F_{tube_1}} \sum_{j_1=1}^{J_1} \gamma_{f_{tube_1}} \delta_{1,j_1} z^{\bar{s}_{f_{tube_1}}, \bar{d}_{f_{tube_1}}}$$

$$= \sum_{f_{tube_1}=1}^{\xi_{tube_1}} \tilde{p}_{f_{tube_1}} z^{\bar{s}_{f_{tube_1}}, \bar{d}_{f_{tube_1}}},$$

where $\bar{s}_{f_{tube_1}}$ is the remaining performance surplus vector that can be obtained using the method depicted in Fig. 2, ξ_{tube_1} represents the number of terms after combining similar

terms, $\tilde{p}_{f_{tubeq}}$ is the probability corresponding to the joint event, and K is the set of thresholds for the components connected by common bus 1.

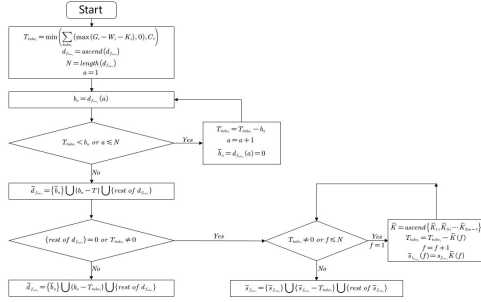


Fig. 2. Performance surplus redistributed method.

In Fig. 2, N is the vector length for the performance deficiency of each component in common bus 1 and $\bar{K}$ is the set for the difference between the performance surplus and threshold of the component whose performance surplus does not satisfy the system threshold.

Similar to that in [9], operator $\odot$ is used to calculate the UGF of $\bar{D}_{tubeq2}$.

$$\bar{U}_{\Omega_2}(z) = U_{\Omega_2}(z) \odot \eta_2(z) \quad (22)$$

$$= \sum_{f_{tubeq2}=1}^{F_{tubeq2}} \sum_{j_2=1}^{J_2} \gamma_{f_{tubeq2}, j_2} \delta_{2, j_2} z^{\bar{d}_{tubeq2}}$$

9) Therefore, using (12) and (13), we obtain the reliability of the system as follows:

$$R = \Pr\{L \geq n\} = \Pr\left\{\sum_{i=1}^m I(\bar{s}_{tubeq1}(i) \geq K(i), \bar{d}_{tubeq2}(i) = 0) \geq n\right\} \quad (23)$$

V. ILLUSTRATIVE EXAMPLE

In this section, the integrated task-processing system of IMA is used to demonstrate the feasibility and validity of the extended model. An integrated task-processing system is primarily responsible for computing and processing data collected from subsystems, such as navigation and communication systems. The integrated task-processing system comprises of GPU1 and GPU2. The GPU consists of a central processing unit composed of three cores and random access memory (RAM). CPU1 and CPU2 are connected by common bus 1, while RAM1 and RAM2 are connected by common bus 2, as depicted in the structure diagram in Fig. 3. The surplus performance, including the data processing ability or memory, can be transferred to a GPU with performance deficiency by the respective common buses. When both GPUs are in a working state, the system is under normal operating condition.

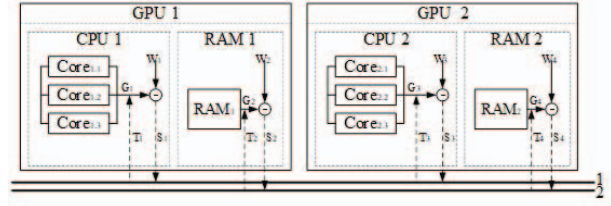


Fig. 3. Structure of integrated task processing system of IMA.

The CPU and RAM exhibit random discrete performance and its corresponding probability and must satisfy the random discrete performance with PMF. The performance levels of each core in the CPUs and the demand level of each CPU with their corresponding occurrence probabilities are listed in Table I.

TABLE I. PERFORMANCES OF THE CORES AND DEMANDS OF THE CPUs WITH CORRESPONDING PROBABILITY

CPU ⁽ⁱ⁾	Core ⁽ⁱ⁾	Performance level ⁽ⁱ⁾	Occurrence probability ⁽ⁱ⁾	Demand level ⁽ⁱ⁾	Occurrence probability ⁽ⁱ⁾
1 ⁽ⁱ⁾	1 ⁽ⁱ⁾	$g_{1,1} = \{3, 2, 0\}$	$\alpha_{1,1} = \{0.7, 0.2, 0.1\}$	$w_1 = \{5, 1\}$	$\beta_1 = \{0.7, 0.3\}$
	2 ⁽ⁱ⁾	$g_{1,2} = \{3, 0\}$	$\alpha_{1,2} = \{0.8, 0.2\}$		
	3 ⁽ⁱ⁾	$g_{1,3} = \{1, 0\}$	$\alpha_{1,3} = \{0.8, 0.2\}$		
2 ⁽ⁱ⁾	1 ⁽ⁱ⁾	$g_{2,1} = \{5, 3, 0\}$	$\alpha_{2,1} = \{0.8, 0.1, 0.1\}$	$w_3 = \{5, 0\}$	$\beta_3 = \{0.9, 0.1\}$
	2 ⁽ⁱ⁾	$g_{2,2} = \{2, 0\}$	$\alpha_{2,2} = \{0.8, 0.2\}$		
	3 ⁽ⁱ⁾	$g_{2,3} = \{1, 0\}$	$\alpha_{2,3} = \{0.9, 0.1\}$		

The performance and demand levels of each RAM with their corresponding occurrence probabilities are listed in Table II.

TABLE II. PERFORMANCE OF RAMS AND RAM DEMAND WITH THEIR CORRESPONDING PROBABILITIES

RAM ⁽ⁱ⁾	Performance level ⁽ⁱ⁾	Occurrence probability ⁽ⁱ⁾	Demand level ⁽ⁱ⁾	Occurrence probability ⁽ⁱ⁾
1 ⁽ⁱ⁾	$g_2 = \{3, 2, 0\}$	$\alpha_2 = \{0.7, 0.2, 0.1\}$	$w_2 = \{2, 1, 0\}$	$\beta_2 = \{0.7, 0.2, 0.1\}$
2 ⁽ⁱ⁾	$g_4 = \{2, 0\}$	$\alpha_4 = \{0.7, 0.3\}$	$w_4 = \{1, 0\}$	$\beta_4 = \{0.8, 0.2\}$

The PMFs of common bus 1 and bus 2 are presented in Table III.

TABLE III. PMFS OF TRANSMITTING ABILITY OF COMMON BUS

Common buses ⁽ⁱ⁾	Transmission ⁽ⁱ⁾	Probability ⁽ⁱ⁾
1 ⁽ⁱ⁾	4 ⁽ⁱ⁾	0.7 ⁽ⁱ⁾
	2 ⁽ⁱ⁾	0.2 ⁽ⁱ⁾
	0 ⁽ⁱ⁾	0.1 ⁽ⁱ⁾
2 ⁽ⁱ⁾	3 ⁽ⁱ⁾	0.8 ⁽ⁱ⁾
	2 ⁽ⁱ⁾	0.1 ⁽ⁱ⁾
	0 ⁽ⁱ⁾	0.1 ⁽ⁱ⁾

1) The UGF is used to represent the CPUs, RAMs, and two common buses, which are formulated as follows:

$$\text{CPU1: } u_{1,1}(z) = 0.7z^3 + 0.2z^2 + 0.1z^0, u_{1,2}(z) = 0.8z^3 + 0.2z^0,$$

$$u_{1,3}(z) = 0.8z^1 + 0.2z^0; \omega_1(z) = 0.7z^5 + 0.3z^1.$$

$$\text{CPU2: } u_{3,1}(z) = 0.8z^5 + 0.1z^3 + 0.1z^0, u_{3,2}(z) = 0.8z^2 + 0.2z^0,$$

$$u_{3,3}(z) = 0.9z^1 + 0.1z^0; \omega_3(z) = 0.9z^5 + 0.1z^0.$$

$$\text{RAM1: } u_2(z) = 0.7z^3 + 0.2z^2 + 0.1z^0;$$

$$\begin{aligned}\omega_2(z) &= 0.7z^2 + 0.2z^1 + 0.1z^0 \\ \text{RAM2: } u_4(z) &= 0.7z^2 + 0.3z^0 \\ \omega_4(z) &= 0.8z^1 + 0.2z^0\end{aligned}$$

The UGFs of the transmitting capacities of the common buses are obtained as:

$$\begin{aligned}\eta_1(z) &= 0.7z^4 + 0.2z^2 + 0.1z^0 \\ \eta_2(z) &= 0.8z^3 + 0.1z^2 + 0.1z^0\end{aligned}$$

2) Using $\oplus$, we calculate the UGF of the performance of CPU1 in terms of $u_{1,1}(z)$, $u_{1,2}(z)$, and $u_{1,3}(z)$ based on (16) as follows:

$$\begin{aligned}u_1(z) &= u_{1,1}(z) \oplus u_{1,2}(z) \oplus u_{1,3}(z) \\ &= 0.448z^7 + 0.24z^6 + 0.032z^5 + 0.176z^4 + 0.076z^3 + \\ &\quad 0.008z^2 + 0.016z + 0.004z^0\end{aligned}$$

3) Similar to step 2), we obtain the UGF representation of the processing quantities of CPU2 and CPU3 as follows:

$$\begin{aligned}u_2(z) &= 0.576z^8 + 0.064z^7 + 0.216z^6 + 0.024z^5 + 0.018z^4 + \\ &\quad 0.074z^3 + 0.008z^2 + 0.018z + 0.002z^0\end{aligned}$$

4) The PMF of S_1 and D_1 in the UGF representation is obtained using the composition operator $\otimes$ over $u_1(z)$ and $\omega_1(z)$:

$$\begin{aligned}\Delta_1(z) &= u_1(z) \otimes \omega_1(z) = 0.1344z^{[6],[0]} + 0.072z^{[5],[0]} + 0.0096z^{[4],[0]} \\ &\quad + 0.0528z^{[3],[0]} + 0.3364z^{[2],[0]} + 0.1704z^{[1],[0]} + 0.0272z^{[0],[0]} + \\ &\quad 0.1244z^{[0],[1]} + 0.0532z^{[0],[2]} + 0.0056z^{[0],[3]} + \\ &\quad 0.0112z^{[0],[4]} + 0.0028z^{[0],[5]}\end{aligned}$$

The UGF of S_2 and D_2 is calculated using the same method and obtained as:

$$\begin{aligned}\Delta_2(z) &= u_2(z) \otimes \omega_2(z) = 0.07z^{[3],[0]} + 0.16z^{[2],[0]} + 0.53z^{[1],[0]} + \\ &\quad 0.15z^{[0],[0]} + 0.02z^{[0],[1]} + 0.07z^{[0],[2]}\end{aligned}$$

The same method is used to obtain the UGF of S_3 and D_3 :

$$\begin{aligned}\Delta_3(z) &= u_3(z) \otimes \omega_3(z) = 0.2304z^{[5],[0]} + 0.0576z^{[4],[0]} + \\ &\quad 0.0256z^{[3],[0]} + 0.4096z^{[2],[0]} + 0.1008z^{[1],[0]} + 0.0448z^{[0],[0]} \\ &\quad 0.0976z^{[0],[1]} + 0.0216z^{[0],[2]} + 0.0096z^{[0],[3]} + 0.0024z^{[0],[4]}\end{aligned}$$

S_4 and D_4 in the UGF representation are obtained as:

$$\begin{aligned}\Delta_4(z) &= u_4(z) \otimes \omega_4(z) = \\ &\quad 0.14z^{[2],[0]} + 0.56z^{[1],[0]} + 0.06z^{[0],[0]} + 0.24z^{[0],[1]}\end{aligned}$$

5) Using (19), we obtain the PMF of S_{tube_2} and D_{tube_2} in the UGF representation as follows:

$$\begin{aligned}U_{\Omega_2} &= \Delta_2(z) \oplus \Delta_4(z) = 0.0098z^{5,[0,0]} + 0.0616z^{4,[0,0]} + 0.168z^{3,[0,0]} \\ &\quad + 0.3274z^{2,[0,0]} + 0.1158z^{1,[0,0]} + 0.009z^{0,[0,0]} + 0.0168z^{3,[0,1]} \\ &\quad + 0.0098z^{2,[2,0]} + 0.0028z^{2,[1,0]} + 0.0392z^{1,[2,0]} + 0.0384z^{2,[0,1]} \\ &\quad + 0.0112z^{1,[1,0]} + 0.1272z^{1,[0,1]} + 0.036z^{0,[0,1]} + 0.0048z^{0,[1,1]} + \\ &\quad 0.0168z^{0,[2,1]} + 0.0012z^{0,[1,0]} + 0.0042z^{0,[2,0]}\end{aligned}$$

6) Using (22), we calculate the UGF of $\bar{S}_{tube_2}$ and $\bar{D}_{tube_2}$ as:

$$\begin{aligned}\bar{U}_{\Omega_2}(z) &= U_{\Omega_2}(z) \odot \eta_2(z) = 0.0098z^{5,[0,0]} + 0.0616z^{4,[0,0]} + \\ &\quad 0.168z^{3,[0,0]} + 0.34252z^{2,[0,0]} + 0.15288z^{1,[0,0]} + 0.14238z^{0,[0,0]} + \\ &\quad 0.00168z^{3,[0,1]} + 0.00098z^{2,[2,0]} + 0.00028z^{2,[1,0]} + 0.0392z^{1,[2,0]} + \\ &\quad 0.00384z^{2,[0,1]} + 0.00112z^{1,[1,0]} + 0.01272z^{1,[0,1]} + 0.036z^{0,[0,1]} + \\ &\quad 0.0048z^{0,[1,1]} + 0.0168z^{0,[2,1]} + 0.0012z^{0,[1,0]} + 0.0042z^{0,[2,0]}\end{aligned}$$

7) The possible situations and corresponding probabilities of the RAM are listed in Table IV.

TABLE IV. POSSIBLE SITUATIONS AND CORRESPONDING PROBABILITIES OF RAM

Scenario ⁽²⁾	RAM ⁽²⁾		Corresponding probability ⁽²⁾
	1 ⁽²⁾	2 ⁽²⁾	
1 ⁽²⁾	work ⁽²⁾	work ⁽²⁾	0.87718 ⁽²⁾
2 ⁽²⁾	work ⁽²⁾	fail ⁽²⁾	0.05424 ⁽²⁾
3 ⁽²⁾	fail ⁽²⁾	work ⁽²⁾	0.04698 ⁽²⁾
4 ⁽²⁾	fail ⁽²⁾	fail ⁽²⁾	0.0216 ⁽²⁾

8) Table V lists the possible scenarios and the corresponding probabilities obtained using (18) and (21).

TABLE V. POSSIBLE SCENARIOS AND CORRESPONDING CPU PROBABILITIES

Scenario ⁽²⁾	CPU ⁽²⁾		Corresponding probability ⁽²⁾
	1 ⁽²⁾	2 ⁽²⁾	
1 ⁽²⁾	work ⁽²⁾	work ⁽²⁾	0.54992 ⁽²⁾
2 ⁽²⁾	work ⁽²⁾	fail ⁽²⁾	0.2221 ⁽²⁾
3 ⁽²⁾	fail ⁽²⁾	work ⁽²⁾	0.11312 ⁽²⁾
4 ⁽²⁾	fail ⁽²⁾	fail ⁽²⁾	0.13185 ⁽²⁾

9) Using (23), we calculate the reliability of the system and determine the reliability of the integrated task-processing system in IMA as 0.4823. The system reliability obtained by assessing the system using a reliability block diagram (RBD) is 0.38. This value is less than that obtained by the extended model because the sharing mechanism is not considered. Therefore, compared to that of the extended model, the RBD result is less accurate.

Finally, we used the proposed model to analyze the tendency of reliability when the number of spare GPUs is increased. Fig. 4 illustrates an increasing tendency of system reliability with increasing number of spare GPU1 when the

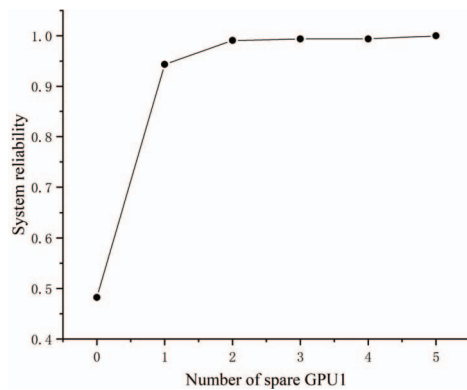


Fig. 4. Trend of system reliability for different numbers of spare GPU1.

number of spare GPU2 is zero.

As shown in Fig. 4, the system reliability first increases rapidly and then gradually approaches a stable value. When the number of spare GPU1 is five, the system reliability is at its maximum value of 0.9998. Considering the cost of the system, it is preferable to use one or three spare GPUs. Under these conditions, the system reliability is 0.9935.

VI. CONCLUSION

In conclusion, this study proposes an extended model for an MSS that considers common shared buses. Based on the extended model, UGF technology was employed to achieve a more accurate and comprehensive assessment of IMA reliability. Taking the integrated task-processing system of IMA as an example, this study investigated the impact of backup GPU quantity on system reliability to demonstrate the accuracy and effectiveness of this method. In future research, we intend to focus on optimizing resource allocation in IMA to provide guidance for the IMA design process.

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