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Technical Note

Scaling laws for two- and three-dimensional wall jet scour based on the phenomenological theory of turbulence

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ABSTRACT

The scour induced by wall jets may cause serious bed erosion and thus damage to hydraulic structures. Previous studies are largely empirical, providing only correlations of experimental data. At present, it is not clear what difference exists in the physical mechanisms of two-dimensional (2D) and three-dimensional (3D) wall jet scour. This study first summarizes previous efforts for predicting wall jet scour depths. Then, a scaling analysis is presented for investigating scour depths by applying the phenomenological theory of turbulence. It is shown that for wall jet cases, the dimensionless equilibrium scour depth can generally be expressed as a power function of the densimetric Froude number and relative roughness height, with the power index depending on jet configurations. The predictions of scour depth using the resulting formulas agree well with published data. The present analysis provides new insights into the understanding of the underlying physical mechanisms of wall jet scour as well as the difference between 2D and 3D configurations.

Keywords: Erosion; local scour; phenomenological theory of turbulence; sediment transport; wall jet

1 Introduction

Local scour around hydraulic structures or natural obstacles plays an important part in morphological changes in fluvial and coastal areas (Deshpande, 2004). Such scour may also pose a risk to the foundation of hydraulic structures, including sluice gates, culverts, bridge piers and weirs, resulting in considerable economic and human losses (Bey et al., 2008; Breusers & Raudkivi, 1991). Therefore, an in-depth understanding of the physical mechanisms of local scour formation is crucial in preventing scour damage in hydraulic and coastal engineering.

One of the common causes of local scour is wall jet, which is characterized by a flow tangential to a channel bed with a velocity higher than that of the ambient fluid. Typical wall jet scour occurs downstream of a sluice gate, culvert or drainage

discharge outlet, etc., and may cause damage to the foundation of relevant hydraulic structures, such as dams and spillways, etc. In terms of the efflux condition, the wall jet can be categorized to be two-dimensional (2D) or three-dimensional (3D), according to the jet expansion ratio, β , defined as the ratio of the channel width B to nozzle width b . As shown in Fig. 1, for a 2D wall jet, $\beta = 1$, and for a 3D wall jet, $\beta \gg 1$. It should be noted that for 2D wall jet scour, an additional criterion should be satisfied, namely, the nozzle width must be considerably greater than the nozzle height to ensure the two-dimensional nature of the flow. On the other hand, there is no definitive threshold value for the jet expansion ratio required for the development of 3D wall jet scour. One commonly used criterion is based on the work by Lim (1995), who suggested that the effect of channel sidewalls can be ignored when β is greater than or equal to 10.

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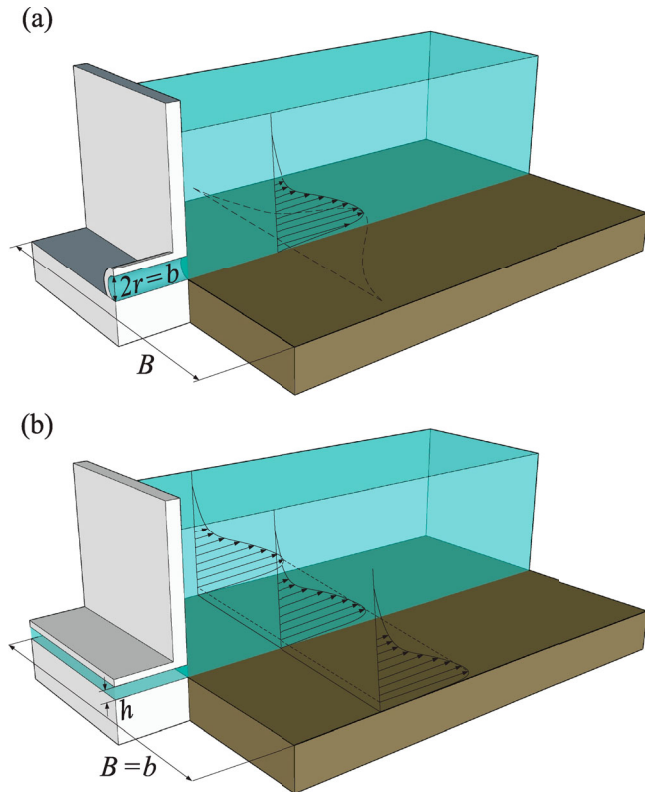


Figure 1 Schematic of (a) three-dimensional (3D) wall jet flow and (b) two-dimensional (2D) wall jet flow

To date, wall jet scour has been extensively investigated for both 2D cases (Aamir & Ahmad, 2016; Kells et al., 2001; Melville & Lim, 2014; Rajaratnam, 1981; Rajaratnam & Macdougall, 1983) and 3D cases (Chiew & Lim, 1996; Rajaratnam & Berry, 1977; S. M. Tan et al., 2020; van der Meulen & Vinje, 1975; Zhao et al., 2019). However, there is no consistent view of the difference between the 2D and 3D cases. Some researchers argued that both scouring processes are similar (Chatterjee et al., 1994; Dey & Sarkar, 2006). Based on the experimental observations, Ali and Lim (1986) derived a unified formula for anticipating the equilibrium scour depth caused by 2D and 3D wall jets. On the other hand, Balachandrar and Reddy (2013) and Schiereck (2004) presented that, for a given efflux velocity, the scour depth subject to 3D wall jets is much smaller than its 2D counterpart as the flow in 3D wall jets expands horizontally and dissipates more energy. To further examine the difference, Table 1 summarizes the scour depth prediction formulas proposed in previous studies. It is worth noting that for the 3D case (Eqs 7–11 in Table 1), the previous studies basically reached a consensus that the dimensionless equilibrium scour depth, $d_{se,3D}/r$ (where $d_{se,3D}$ is the equilibrium depth of 3D wall jet scour and r is the nozzle radius) is linearly related to the densimetric Froude number, F_{dx} . Here, $F_{dx} = U/\sqrt{\Delta g d_x} = \text{densimetric Froude number}$; U = bulk-averaged velocity of the efflux; $\Delta = (\rho_s - \rho)/\rho$; ρ = fluid density; ρ_s = particle density; g = gravitational acceleration; and d_x = particle size for which $x\%$ of sediment particles is

finer. Different x -values have been used in previous studies to take sediment gradation into account (S. M. Tan et al., 2020).

However, the above-mentioned linear relationship may not hold for the case of 2D wall jet. From Table 1, it can be seen that for the 2D case, the dimensionless equilibrium scour depth $d_{se,2D}/h$ (where $d_{se,2D}$ is the equilibrium depth of the 2D case and h is the nozzle height) can be formulated as a power function of F_{dx} , but the power index varies from 0.94 to 2. For example, the equations proposed by Chatterjee et al. (1994) and Dey and Sarkar (2006) show that $d_{se,2D}/h \sim F_{dx}$, which is similar to that for 3D wall jet scour. Meanwhile, Altinbilek and Basmaci (1973) and Lim and Yu (2002) indicated that $d_{se,2D}/h$ scales with $F_{dx}^{3/2}$. Assuming that the cross-sectional average velocity at the location of the maximum scour depth is equal to the critical flow velocity for incipient sediment motion, Breusers and Raudkivi (1991) obtained $d_{se,2D}/h \sim F_{dx}^2$. At present, what the power index should be taken for the 2D wall jet case is still inconclusive. In addition to F_{dx} , another factor to note is the relative roughness height, the ratio of particle diameter to nozzle radius or nozzle height. As shown in Table 1, the relative roughness height is included in most formulas (Eqs 1–11) for the 2D case, but not for the 3D case. Consequently, the discrepancy of the power index and the effect of relative roughness height for the 2D and 3D wall jet scour may imply different scouring mechanisms in the 2D and 3D cases, which have not yet been illuminated before. This is probably because previous formulas were derived empirically from data correlations, which lack a theoretical basis.

The objective of this study is to investigate the differences between 2D and 3D wall jet scour by performing a scaling analysis based on the phenomenological theory of turbulence (PTT). Several prior investigations have successfully applied the PTT to local scour problems (Bombardelli et al., 2018; Bombardelli & Gioia, 2006; Coscarella et al., 2020; Di Nardi et al., 2022; Gioia & Bombardelli, 2005; Manes & Brocchini, 2015). In particular, Gioia and Bombardelli (2005) theoretically derived a formula for predicting the equilibrium scour depth for the 2D impinging jet scour. Bombardelli and Gioia (2006) and Bombardelli et al. (2018) reported that 3D impinging jet scour features different characteristics compared to the 2D case. Although there are inherent differences in the mechanisms of impinging jet and wall jet, they also share some similarities. Therefore, it is important to take into account both differences and similarities when applying the findings from previous studies to wall jets related scour problems.

In the present study, we will extend the fundamental reasoning from PTT to differentiate between 2D and 3D wall jet scour in terms of scaling laws. The rest of this paper is organized as follows: Section 2 presents the derivation of an analytical model; Section 3 validates the model using the published experimental data; Section 4 discusses the different scaling laws for 2D and 3D wall jet scour; and Section 5 concludes the paper.

Table 1 Summary of wall jet scour depth equations available in the literature

Investigator	Equation	Scaling	Number
2D cases			
Chatterjee et al. (1994)	$\frac{d_{se,2D}}{h} = 0.775F_{d50} \left(\frac{d_{50}}{h}\right)^{1/2} \Delta^{1/2}$	$\sim F_{d50}$	(1)
Dey and Sarkar (2006)	$\frac{d_{se,2D}}{h} = 2.59F_{d50}^{0.94} \left(\frac{L_t}{h}\right)^{0.16} \left(\frac{L_a}{h}\right)^{-0.37} \left(\frac{d_{50}}{h}\right)^{1/4}$	$\sim F_{d50}^{0.94}$	(2)
Ali and Lim (1986)	$\frac{d_{se,2D}}{h} = \left[2.3F_{d50}^{3/4} \left(\frac{U}{\omega}\right)^{1/2} \left(\frac{d_{50}}{h}\right)^{0.375} - 1.19 \right]$	$\sim F_{d50}^{5/4}$	(3)
Lim and Yu (2002)	$\frac{d_{se,2D}}{h} = 1.04F_{d50}^{1.47} \sigma_g^{-0.69} \left(\frac{d_{50}}{h}\right)^{0.33} K_L$	$\sim F_{d50}^{1.47}$	(4)
Altinbilek and Basmaci (1973)	$\frac{d_{se,2D}}{h} = \tan\phi^{1/2} \left(\frac{d_{50}}{h}\right)^{1/2} F_{d50}^{3/2}$	$\sim F_{d50}^{3/2}$	(5)
Breusers and Raudkivi (1991)	$\frac{d_{se,2D}}{h} = 0.008 \left(\frac{U}{u_{*c}}\right)^2$	$\sim F_{d50}^2$	(6)
3D cases			
Rajaratnam and Diebel (1981)	$\frac{d_{se,3D}}{r} = [0.41F_{d50} - 0.067]$	$\sim F_{d50}$	(7)
Breusers and Raudkivi (1991)	$\frac{d_{se,3D}}{r} = 0.16 \frac{U}{u_{*c}}$	$\sim F_{d50}$	(8)
Lim (1995)	$\frac{d_{se,3D}}{r} = 0.45F_{d50} \quad (1 \leq F_d \leq 10)$	$\sim F_{d50}$	(9)
Zhao et al. (2019)	$\frac{d_{se,3D}}{r} = 1.03F_{d84} (e^{-1.9(K_s-1.2)^2} + 1)$	$\sim F_{d84}$	(10)
S. M. Tan et al. (2020)	$\frac{d_{se,3D}}{r} = 1.28F_{d84}$	$\sim F_{d84}$	(11)

Note: $F = U/\sqrt{gh}$ = jet Froude number; $F_{dx} = U/\sqrt{\Delta g d_x}$ = densimetric Froude number; U = bulk-averaged velocity of the efflux; $\Delta = (\rho_s - \rho)/\rho$; ρ = fluid density; ρ_s = particle density; g = gravitational acceleration; d_x = particle size for which $x\%$ of sediment particles is finer; $F_d = F_{d50}$; ϕ = angle of repose; ω = settling velocity; u_{*c} = critical shear velocity; K_L = apron collection factor; σ_g = sediment gradation; L_t = tail-water height; L_a = apron length; K_s = ratio of sediment size to pipe radius. $u_{*c} \sim w \sim \sqrt{\Delta g d_{50}}$ for high Reynolds number (Cheng, 2008; Yang, 1973).

2 Methods

During the initial stages of wall jet scour, the scour hole undergoes rapid development. In the scour hole, the flow velocity decreases gradually in the downstream direction with increasing flow depth associated with increasing scour depth. Once the scour process reaches equilibrium, sediment particles located at the bottom of the scour hole become more difficult to entrain. This equilibrium stage can be considered critical, as the flow conditions are comparable to those necessary for incipient sediment motion (Carstens, 1966). If the flow within the scour hole is hydrodynamically rough, the force balance for a bed particle at the bottom of scour hole can be expressed as:

$$\tau = \tau_c \sim (\rho_s - \rho)gd_{50} \quad (12)$$

where τ is the bed shear stress induced by the flow at the bottom of the equilibrium scour hole and τ_c is the critical value for incipient sediment motion. In Eq. (12), the bed slope at the bottom of the scour hole is not considered. This is because experimental observations (Carstens, 1966; Raudkivi, 1998) have shown that the bottom is almost horizontal at the equilibrium stage. Since τ at the bottom of the scour hole decreases with increasing scour depth, it is related not only to the characteristics of the jet flow, but also the scour depth. In the following, τ is formulated by applying the theoretical reasonings from the PTT (Bombardelli et al., 2018; Gioia & Bombardelli, 2001; Manes & Brocchini, 2015).

Assuming that the flow within the scour hole is fully turbulent, the viscous effect can be neglected, and thus $\tau = -\rho \overline{u'w'}$, where u' and w' are the streamwise and vertical velocity fluctuation, respectively, and the overbar denotes the temporal average. Generally, the velocity fluctuation is associated with the motion of eddies of different sizes. Gioia and Bombardelli (2001) argued that u' can be influenced by eddies of all scales, but the contribution from small eddies is negligible. As a result, one gets $u' \sim V$, where V is the characteristic velocity of large eddies that dominates the turbulent kinetic energy (TKE) production. Gioia and Bombardelli (2001) also stated that for the flow over a sediment bed with particle diameter d_{50} , the principal contribution for w' is from the eddies with a size comparable to d_{50} . This suggests that w' scales with the characteristic velocity v_d associated with the eddies of size d_{50} , i.e. $w' \sim v_d$. With the above consideration, τ can be estimated as:

$$\tau = -\rho \overline{u'w'} \sim \rho V v_d \quad (13)$$

According to Kolmogorov (1941), if the TKE is produced at a rate ϵ , it is transferred from large to small eddies at the same rate in the inertial range of turbulence. This yields the link between V and v_d , i.e. $\epsilon \sim V^3/R \sim v_d^3/d_{50}$, where R is the length scale of large eddies. Rearranging it, one gets:

$$v_d \sim V(d_{50}/R)^{1/3} \quad (14)$$

Substituting Eq. (14) into Eq. (13):

$$\tau \sim \rho V^2 (d_{50}/R)^{1/3} \quad (15)$$

Furthermore, at the equilibrium scour stage, the bed shear stress acting on the particles is equal to its critical value, which scales with the submerged weight of particles (Eq. 12). Thus, by combining Eqs (12) and (15), one gets:

$$\rho V^2 (d_{50}/R)^{1/3} \sim (\rho_s - \rho) g d_{50} \quad (16)$$

Equation (16) describes the scaling relationship between the jet flow and sediment particle properties. Next, it will be applied to derive an expression for predicting the equilibrium scour depth. To this end, two key variables in Eq. (16), namely, the characteristic length R and velocity V of the large eddies, need to be evaluated. To simplify analysis, several factors are not considered, including apron length, tail-water depth and sediment gradation. In addition, the 3D wall jet is only limited to the case issuing from a circular nozzle.

2.1 Characteristic length of large eddies

To describe the length scale of large eddies associated with jet flows, one can use the hydraulic radius at the nozzle brink, which is defined as:

$$R = A/\chi \quad (17)$$

where A is the cross-sectional area of the nozzle and χ is the wetted perimeter of the nozzle. Considering the difference between the cross sections of 2D and 3D nozzles (Fig. 1), Eq. (17) can be further rewritten as:

$$R = \begin{cases} h/2 & \text{for 2D wall jet} \\ r/2 & \text{for 3D wall jet} \end{cases} \quad (18)$$

As a length scale, the hydraulic radius can be linked to the size of dominant eddies in various turbulent flows (Cheng et al., 2021; Cheng & Wei, 2019; Gioia & Bombardelli, 2001; Lu et al., 2021; S. M. Tan et al., 2020). For instance, Gioia and Bombardelli (2001) provided a clear explanation of the equivalence between the hydraulic radius and the size of large eddies in open-channel flows. Cheng and Wei (2019) reported that in comparison with other length scales, the pier-related hydraulic radius is more effective in scaling the equilibrium scour depth. Cheng et al. (2021) further extended the concept of the pier hydraulic radius to local scour associated with abutments, culverts and wall jets. These studies have provided compelling evidence supporting the use of the hydraulic radius as an effective length scale for quantifying dominant eddy sizes in turbulent flows.

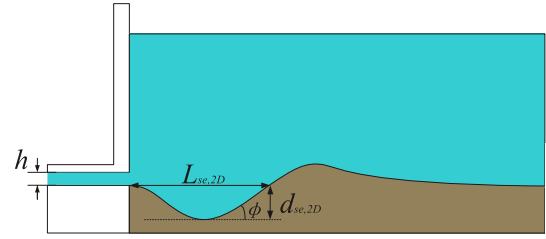


Figure 2 Schematic of the side view of 2D wall jet scour

2.2 Characteristic velocity of large eddies

Recalling Eq. (16), to determine the characteristic velocity V , one may start with the evaluation of the turbulent kinetic energy (TKE) induced by the wall jet. For the sake of simplicity, 2D wall jet scour is considered first. At the equilibrium stage, the TKE produced in the scour hole equals the kinetic energy injected by the jet, so the associate power P can be expressed as:

$$P = \frac{1}{2} \frac{\rho B h U^2 \delta L}{\delta t} = \frac{1}{2} \rho B h U^3 \quad (19)$$

where δL is the length of the jet entering the ambient fluid in time δt . Noting that the dissipation rate is equal to the production rate, $\epsilon \sim P/M$, where M is the fluid mass in the scour hole. Since M is the major component on which the jet flow works, it can be approximated to be:

$$M \sim \rho B L_{se,2D} d_{se,2D} \quad (20)$$

where $L_{se,2D}$ is the length of the equilibrium scour (Fig. 2). At the equilibrium stage, the slope angle of the scour hole is close to the angle of repose (Altinbilek & Basmaci, 1973; Carstens, 1966). Thus, $L_{se,2D} = 2d_{se,2D}/\tan\phi$, and combining Eqs (19) and (20), one gets:

$$\epsilon \sim \tan\phi h U^3 / d_{se,2D}^2 \quad (21)$$

Since $\epsilon \sim V^3/R$ and $R \sim h$, the expression of the characteristic velocity of large eddies for the 2D case can be derived from Eq. (21):

$$V_{2D} \sim (\tan\phi)^{1/3} (h/d_{se,2D})^{2/3} U \quad (22)$$

where subscript 2D denotes the 2D wall jet scour. Equation (22) shows that the characteristic velocity of large eddies is dominated not only by the jet flow velocity U , but also the ratio of the characteristic length of large eddies to equilibrium scour depth $h/d_{se,2D}$.

In comparison with the scour induced by 2D wall jet, the kinetic energy of 3D wall jet will be dissipated in all three dimensions, which leads to a different dissipation rate of TKE. As shown in Fig. 1, the 3D wall jet is subjected to an additional lateral mixing with the ambient fluid and therefore causes more energy dissipation compared to its 2D counterpart. For the 3D wall jet case, Eqs (19) and (20), respectively, can be

rewritten as $P = \rho \pi r^2 U^3 / 2$ and $M \sim \rho d_{se,3D}^3 / (\tan \phi)^2$. Thus, the associated dissipation rate can be expressed as:

$$\epsilon \sim (\tan \phi)^2 r^2 U^3 / d_{se,3D}^3 \quad (23)$$

Recalling that $\epsilon \sim V_{3D}^3 / r$, and one gets:

$$V_{3D} \sim (\tan \phi)^{2/3} (r / d_{se,3D}) U \quad (24)$$

where V_{3D} is the characteristic velocity of large eddies induced by a 3D wall jet. Equations (22) and (24) indicate that the characteristic velocity of large eddies, for either 2D or 3D wall jet scour, is closely related to the equilibrium scour depth.

2.3 Equilibrium scour depth

The preceding two sections show how to derive the scaling expressions of the characteristic length and velocity associated with the two types of wall jets. Noting that the bed shear stress can be evaluated by V and R (Eq. 15), and at the equilibrium stage, the bed shear stress scales with the submerged weight of particles (Eq. 12), thus the equilibrium scour depth can be expressed as:

$$d_{se,2D}/h \sim (\tan \phi)^{1/2} (d_{50}/h)^{1/4} F_{d50}^{3/2} \quad (25)$$

and

$$d_{se,3D}/r \sim (\tan \phi)^{2/3} (d_{50}/r)^{1/6} F_{d50} \quad (26)$$

where $F_{d50} = U / \sqrt{\Delta g d_{50}}$. It is interesting to note that the functional relationships, which have forms similar to Eqs (25) and (26), have been reported in previous studies, but they are all empirical. For example, Altinbilek and Basmaci (1973) obtained their empirical equation (Eq. 5) solely through curve fitting. Except for the different power indexes of d_{50}/h , Eq. (5) has the same form as Eq. (25). In addition, the empirical equation proposed by Lim and Yu (2002) also suggested that $d_{se,2D}/h$ nearly scales with $F_{d50}^{3/2}$ (Eq. 4). As for 3D wall jet scour occurring over a uniform-sediment bed, the previous studies generally confirm the linear relationship of $d_{se,3D} \sim F_{d50}$ (Table 1).

Furthermore, if the scaling relations in Eqs (25) and (26) are approximated to be linear (Gioia & Bombardelli, 2005), and noting that the angle of repose usually varies in a limited range (Cheng & Zhao, 2017), one gets:

$$d_{se,2D}/h = c_1 (d_{50}/h)^{1/4} F_{d50}^{3/2} \quad (27)$$

and

$$d_{se,3D}/r = c_2 (d_{50}/r)^{1/6} F_{d50} \quad (28)$$

where c_1 and c_2 are constants. From Eqs (27) and (28), it can be seen that the equilibrium scour depths for 2D and 3D wall jets follow different scaling laws. In the equations, the relative

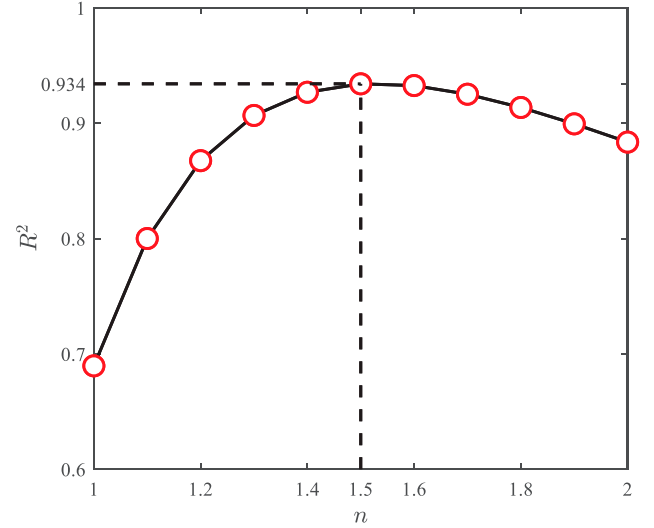


Figure 3 R^2 values versus different power indexes

roughness height, i.e. d_{50}/h or d_{50}/r , represents the size ratio of small to large eddies, which also affects the equilibrium scour depth. This will be further discussed in Section 4.

3 Results

In this section, the proposed models will be validated with the published laboratory data, as tabulated in Table 2. The data are selected under the conditions that the effects of tail-water depth, apron length, downstream channel width, and sediment gradation are negligible.

As summarized in Table 1, for 2D wall jet scour, previous formulas show that $d_{se,2D}/h \sim [F_{d50} (d_{50}/h)^{1/6}]^n$, where n varies from 0.94 to 2. To quantitatively assess the performance of the theoretical model, the coefficient of determination (R^2) is calculated. The results of R^2 vary with the power index n , as shown in Fig. 3. Using the data summarized and applying regression analysis, one can get the best fit when $n = 3/2$ (Fig. 3). This empirical n -value turns out to be consistent with the power index derived from the theoretical model (Eq. 27).

To evaluate the coefficient c_1 in Eq. (34), Fig. 4 presents the correlation between the dimensionless equilibrium scour depth with the densimetric Froude number and relative roughness height. It shows that Eq. (27) with $c_1 = 0.87$ provides the best description of the data trend ($R^2 = 0.93$). Therefore, Eq. (27) can be revised to be:

$$d_{se,2D}/h = 0.87 F_{d50}^{3/2} (d_{50}/h)^{1/4} \quad (29)$$

Figure 5 shows the variation of $d_{se,3D}/r$ with the combined parameter $F_{d50} (d_{50}/r)^{1/6}$ for the 3D wall jet case. The best correlation occurs when $c_2 = 0.71$ with $R^2 = 0.92$. Consequently, Eq. (28) can be rewritten as:

$$d_{se,3D}/r = 0.71 F_{d50} (d_{50}/r)^{1/6} \quad (30)$$

Table 2 Summary of experimental data

Investigator	Jet type	d_{50} (mm)	σ_g	h or r (mm)	U (m/s)	d_{se} (m)	No. of runs
Rajaratnam (1981)	2D	1.20–2.38	1.20	3.6–25	0.87–2.22	0.04–0.20	14
Mazurek et al. (2009)	2D	2.08	1.20	25	0.27–1.13	0.03–0.20	5
Gautam (2008)	2D	1.32–2.20	1.32–1.20	16	0.85–1.35	0.07–0.13	15
Kells et al. (2001)	2D	0.57–2.18	1.28–1.45	10	0.56–0.97	0.07–0.14	18
Y. P. Tan (2003)	3D	1.60	1.13	12.7	1.16–2.5	0.06–0.12	6
Pee (2016)	3D	0.77	1.16	4–10	0.60–2.10	0.02–0.05	19
Seah (1997)	3D	8.7	1.11	20	1.35–4.47	0.05–0.14	26
Salam (2016)	3D	0.5	1.45	13.7	0.73–1.26	0.05–0.09	16
S. M. Tan et al. (2020)	3D	0.25–4.90	1.17–1.44	7.5–12.7	1.01–3.35	0.04–0.11	20
Lim and Chin (1993)	3D	1.65	1.25	7.5	0.79–4.55	0.01–0.14	6
Chiew and Lim (1996)	3D	0.25–1.65	1.25–1.44	6.4–12.7	0.53–5.43	0.01–0.20	33
Rajaratnam and Diebel (1981)	3D	1.05	1.5	6.4–7.5	0.45–2.32	0.03–0.08	17
Ade and Rajaratnam (1998)	3D	0.24	1.46	2.5	4.3–5.5	0.07–0.10	3

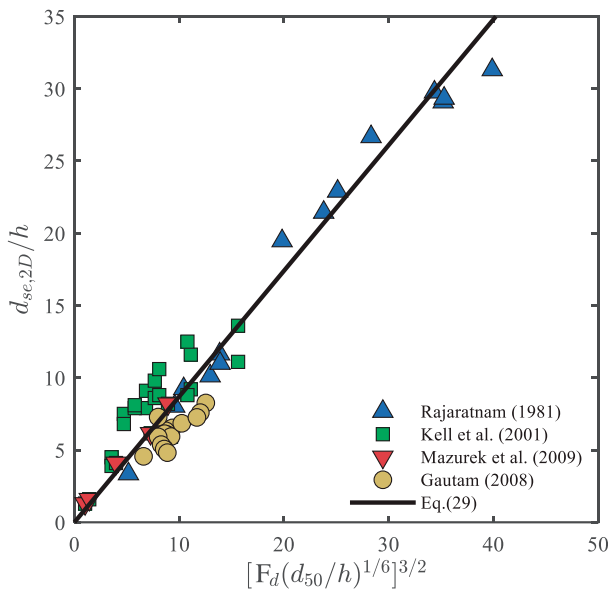


Figure 4 Variation of the dimensionless equilibrium scour depth with densimetric Froude number and the relative roughness height for 2D wall jet

Comparing the relative roughness height terms in Eqs (29) and (30), it can be observed that the power index of $1/6$ for 3D wall jet scour is smaller than the index of $1/4$ for 2D wall jet scour, indicating that 3D wall jet scour is less affected by relative roughness height. As shown in Fig. 6, for a given F_{d50} , the term of relative roughness height in 3D wall jet scour is closer to 1 than that in 2D case. This finding may justify the fact that most empirical formulas for 3D wall jet scour ignore the effect of relative roughness height.

4 Discussion

Equations (25) and (26) show that the equilibrium scour depth, when normalized by the characteristic length of large eddies, follows different scaling laws for 2D and 3D wall jet scour. To further explore the difference, using R to replace h and r , Eqs

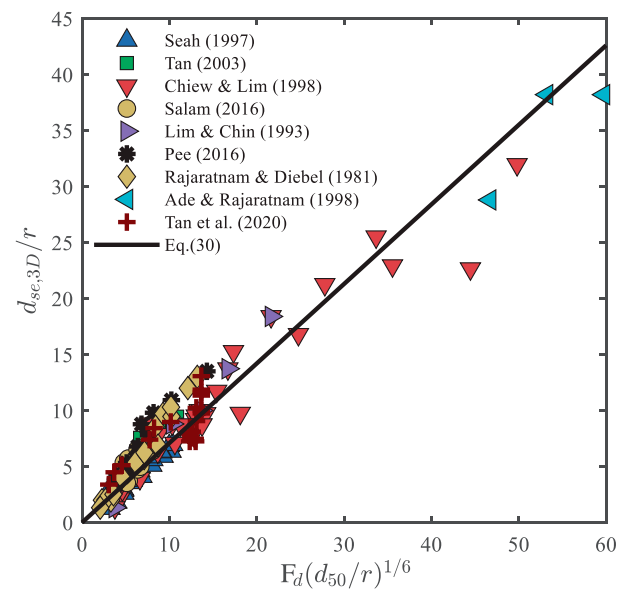


Figure 5 Variation of the dimensionless equilibrium scour depth with densimetric Froude number and the relative roughness height for 3D wall jet

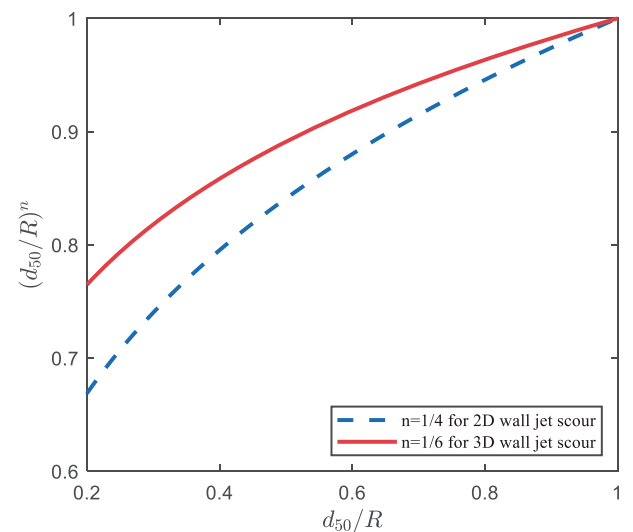


Figure 6 Different effects of the relative roughness height on 2D and 3D wall jet scour

(25) and (26) can be expressed in a unified form, i.e.:

$$\frac{d_{se}}{R} = \begin{cases} 0.87 \left(\frac{U(d_{50}/R)^{1/6}}{\sqrt{\Delta g d_{50}}} \right)^{3/2} & \text{for 2D wall jet} \\ 0.71 \frac{U(d_{50}/R)^{1/6}}{\sqrt{\Delta g d_{50}}} & \text{for 3D wall jet} \end{cases} \quad (31)$$

In Eq. (31), $U(d_{50}/R)^{1/6}$ can be equivalently considered as a friction velocity u_* in an analogous sense. This is because according to the Manning–Strickler formula, the ratio of friction velocity to average velocity can be expressed as $u_* \sim U(d_{50}/R)^{1/6}$ (Yen, 1992). Therefore, $U(d_{50}/R)^{1/6}$ can be regarded as a measure of the shear force intensity of a wall jet, albeit not an exact quantification of the shear force working at the bottom of the scour hole. By noting that the Shields number represents the ratio of the average bed shear stress to the submerged weight of sediment particles per unit bed area, one may define a similar dimensionless parameter, to be referred as “jet Shields number”, i.e.:

$$\Theta_j = \frac{U^2 (d_{50}/R)^{1/3}}{\Delta g d_{50}} \quad (32)$$

which quantifies the strength of the jet-induced bed shear stress relative to the particle weight. In terms of Θ_j , Eq. (31) can be simplified as:

$$\frac{d_{se}}{R} = \begin{cases} 0.87 \theta_{j,2D}^{3/4} \\ 0.71 \theta_{j,3D}^{1/2} \end{cases} \quad (33)$$

As for the change in the power index in Eq. (33), it can be explained by the turbulence dissipation rate (Eqs 21, 23). The lateral mixing and diffusion in the 3D case result in a reduced contribution of jet flow energy to the excavation of the scour hole, in comparison to the 2D case.

In addition, it should be mentioned that although the jet Shields number has considered the effect of relative roughness height in the form, it holds only in the condition that the small eddy size d_{50} is in the inertial subrange, i.e. $\eta \ll d_{50} \ll R$, where η is the Kolmogorov length (Bombardelli & Gioia, 2006; Gioia & Bombardelli, 2005; Lu et al., 2021).

5 Conclusions

In this study, theoretical models are derived for predicting scour depth under 2D and 3D wall jet conditions, allowing for a clear illustration of the differences between the two. The following conclusions can be drawn:

- (i) In light of the phenomenological theory of turbulence, different scaling laws are developed for 2D and 3D wall jet scour. The reliability of the proposed models has been validated with the published experimental data.
- (ii) The proposed models indicate that the dimensionless equilibrium scour depth can be formulated as a power function

of the newly-defined jet Shields number. The power index is 3/4 for 2D wall jet and 1/2 for 3D wall jet.

- (iii) The difference between 2D and 3D wall jet scour is attributed to their dissimilar turbulence dissipation rates. Compared to the 3D case, the 2D wall jet contains more turbulent energy per unit volume in the scour hole for a given efflux velocity, resulting in more severe local scour.

Although the proposed models provide a solid physical background to empirical equations, their application is limited due to the neglect of several effects that should be carefully considered in future studies, which may require high-resolution experimental measurements and numerical simulations. In addition, the further examination of the potential scale effect is expected to be made with more field observations and large-scale experiments.

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Notation

- $d_{se,2D}$ = equilibrium scour depth of 2D wall jet scour (m)
 $d_{se,3D}$ = equilibrium scour depth of 3D wall jet scour (m)
 d_x = particle size for which x% of sediment particles is finer (m)
 u_{*c} = critical shear velocity (m s^{-1})
 v_d = characteristic velocity of eddies of size d_{50} (m s^{-1})
 ρ_s = particle density (kg m^{-3})
 B = channel width (m)
 b = nozzle width (m)
 F = jet Froude number (–)
 F_{dx} = densimetric Froude number (–)
 g = gravitational acceleration (m s^{-2})
 h = nozzle height (m)
 L = length of the equilibrium scour (m)
 M = fluid mass in the scour hole
 P = associate power of the TKE production (W)
 r = nozzle radius (m)
 R = length scale of large eddies (m)
 U = bulk-averaged velocity of the efflux (m s^{-1})
 V = characteristic velocity of large eddies (m s^{-1})
 ϵ = TKE dissipation rate ($\text{m}^2 \text{s}^{-1}$)
 Θ_j = jet Shields number (–)
 β = jet expansion ratio (–)

- ν = fluid kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
 ρ = fluid density (kg m^{-3})
 τ = bed shear stress (N m^{-2})
 φ = angle of repose ($^\circ$)
 σ_g = sediment gradation (—)

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