

A typology of Marxian transformation procedures with endogenous exploitation rate

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Abstract

This paper constructs a unified framework to evaluate the Marxian transformation models, where the exploitation rate is an endogenous variable determined within value and price of production systems. All different procedures are based on two different value-price invariance equations. These could be chosen among either two aggregate product equations (gross or net output) or three capital invariant equations (total, variable or constant). Different combinations of invariance equations result in most Marxian solutions developed in recent decades. The solution does not imply that one model (prices) is logically prior to the other (values), but that in fact that the joint solution is needed.

KEYWORDS

labor values, Marxian model, prices of production, Sraffian model, surplus value, transformation problem

JEL CLASSIFICATION

B14, B24

1 | INTRODUCTION

The so-called Marxian transformation problem is at the core of the labor value theory. This relates to the conceptual and analytical map to transform labor values into prices of production. The problem arises as some value and price aggregates cannot hold simultaneously (see, for instance, Mohun, 1994 and Mohun and Veneziani, 2017). The Sraffian model (Sraffa, 1960) appears as an alternative consistent scheme of prices of production where prices, wages and the profit rate can be obtained without reference to the labor value system (see, e.g., Steedman, 1977), except for the fact

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that exploitation is necessary for positive profits (a property defined as the Fundamental Marxian Theorem, Morishima, 1973).

This paper constructs a typology of the transformation problem solutions for the case where exploitation, surplus value and necessary labor are determined *endogenously* from a Sraffian prices of production simple reproduction scheme. Endogenous here means that the solution is not based on a subsistence or real wage bundle (as in the standard neo-Ricardian approach to the transformation problem), but it is solved jointly with the price variables. The analysis presented here is framed within Mohun and Veneziani's (2017) analysis of a classical long-period analysis and the dualist approach, in which there is both a value system and a price system. All the solutions are based on two different invariance equations, each of which equalizes two aggregates, one in values to another in prices. Each of the invariance conditions contains one price aggregate magnitude and a value aggregate magnitude on each side of the equality. Following Mohun and Veneziani's (2017) framework these equations are implicitly defining the value of money, units of labor time per unit of money, such that both sides of the equalities are expressed on the same measure. These could be chosen among either two aggregate product equations (gross or net output) or three capital invariant equations (total, variable or constant). Different combinations of invariance equations result in different Marxian solutions in the dualist approach. As a result, this paper proposes to frame the different solutions to the transformation problem in a unified framework that uses the value and price systems.

The proposed model follows Garegnani (1984) who claims that Sraffa-type equations strengthen Marx's model. This paper proposes to evaluate different so-called solutions of the transformation problem in terms of two specific dimensions. First, in terms of the Fundamental Marxian Theorem (FMT1) of Morishima (1973) that relates the profit rate to the surplus value. In a weak form, it states that there is profit only if there is exploitation. Second, in terms of an alternative comparison of value and price systems in terms of the necessary labor and wage, which will be defined as FMT2. In a weak form FMT2 would state that the necessary labor is positive only if the wage is positive.¹

Moreover, the studied framework also explores if the Marxian system strengthens the Sraffian one, by proposing that the wage-profit curve indeterminacy in Sraffian models can be explained by the use of the additional labor value category from a joint model. The solution does not imply that one model (prices) is logically prior to the other (values), but that in fact that the joint solution is required for determining all the variables. This approach is thus motivated by Marx's original program of explaining profit rates as a derivation of labor exploitation, and as such, it suggests that Marxian model also strengthens the Sraffian model. This is because we can focus on labor value categories (i.e. surplus rate) to explain variation along the wage-profit curve in the Sraffian framework.

The main idea to treat the Marxian exploitation rate or surplus value as an endogenous parameter coming from prices is taken from the 'New Interpretation' (NI), see, for instance, Foley (1982, 1986, 2000), Duménil (1983), Glick and Ehrbar (1987) and other related papers. The central feature in the NI is that workers in a capitalist society are paid in money wages, and thus exploitation and surplus value should be analyzed in terms of the proportion of value added (in prices) that goes to workers. The NI is a special case where the value-price invariance in variable capital and in net output are used. It should be noted, however, that the NI is not a dualist approach.

Following the spirit of the NI, we argue that distributive value categories are not given a priori, and they need to be deduced ex post. Nevertheless, the NI is only one possible transformation procedure.² Other frameworks, such as Loranger (2004), are specifically designed to obtain an equivalence in the profit rate, although at a cost of having an endogenous wage rate. As noted in Montes-Rojas (2017)

¹I am grateful to an anonymous referee for this conceptualization.

²See Moseley (2000) and Fine, Lapavistas and Saad-Filho (2004) for a critical discussion of the NI solution.

this is in fact the result of two invariance equations based on gross output and total capital. Wolff, Roberts and Callari (1982) postulate the value-price invariance of constant capital in labor values and prices. Montes-Rojas (2017) evaluates a capital invariant solution based on both constant and variable capital invariance equations.

Eatwell (1975), Roberts (2009) and Araujo (2019) among others study the particular case of the use of the Sraffa's Standard Commodity where the equivalence of both price and value systems is maintained. Their specific model is based on the neo-Ricardian framework using a worker's consumption bundle. We argue that this is yet another potential solution based on the list of invariance conditions above, although it produces two independent systems where only with production based on a given proportionality (that of the Standard Commodity) can the solution be obtained without incompatibilities.³

The rest of the paper is organized as follows. Section 2 presents the main analytical framework. Section 3 proposes a set-up for the price and value solutions, while Section 4 explores the analytical derivations of all the cases involved. Section 5 presents a summary of the typology, while Section 6 applies the latter to a numerical example. Section 7 concludes.

2 | A UNIFIED VALUE AND PRICE FRAMEWORK

2.1 | Labor value system

An economy is characterized by a Leontief input-output $n \times n$ matrix, A with elements $a_{ij} \geq 0$, $i, j = 1, \dots, n$, that specifies the quantity of commodity i required to produce one unit of good j , and a $1 \times n$ vector $\ell \gg 0$ specifying labor units required to produce one unit of each commodity, ℓ_i , $i = 1, \dots, n$. We assume that A is indecomposable and thus $(I - A)^{-1}$ is positive, and the system is productive.

The labor values are given by a $1 \times n$ vector $\lambda \gg 0$ that solves the following equation,

$$\lambda A + \ell = \lambda, \quad (1)$$

that is,

$$\lambda = \ell(I - A)^{-1}. \quad (2)$$

A key feature in Marxian labor value theory is the surplus value rate. This determines the ratio of the amount of value that is appropriated by capitalists as opposed to what is paid (in value) to workers. For the vector of labor components, ℓ , we will decompose it in $(1 - \alpha + \alpha)\ell$ for $\alpha \in [0, 1]$, where α is the portion of labor that is appropriated by capitalists (defined as the *appropriation rate* here for convenience) and $1 - \alpha$ is the portion of labor that goes to the workers. Thus, we have $\alpha\ell$ is the vector of surplus labor and $(1 - \alpha)\ell$ is the vector of necessary labor. Then, we have $\sigma = \alpha/(1 - \alpha)$ as the surplus value rate. A central feature of the analysis in this paper is that α is an endogenous parameter (and therefore σ also is) to be determined within the model, and jointly with the Sraffian prices of production system. That is, this rate will be determined ex post, together with prices and distributive

³As argued by Sinha (2000), the Standard Commodity measure shifts the study from the appropriation of labor to the distribution of net product. See also the discussion in Patniak (1989).

variables.⁴ This approach is in contrast to the neo-Ricardian approach where this rate is implicitly determined by the value of a given bundle of goods. Thus, we use the following equation,

$$\lambda \mathbf{A} + (1 - \alpha + \alpha)\ell = \lambda. \quad (3)$$

Using the terminology of Pasinetti (1977) the labor value system will be defined as the Marxian system.

2.2 | Price system

Consider now the price system, the so-called prices of production framework, which corresponds to the long-period analysis (see Kurz and Salvadori, 1995). We will denote this model as the Sraffian framework. Let these prices be given by a $1 \times n$ vector $\mathbf{p}$, together with a uniform profit rate r and wage w (wages are advanced to workers rather than paid after production), such that

$$(\mathbf{p}\mathbf{A} + w\ell)(1 + r) = \mathbf{p}. \quad (4)$$

For positive price solution, $\mathbf{p} \gg 0$, the profit rate r lies in the segment $[0, R]$ where $1/(1 + R)$ is the maximum left eigenvalue of the matrix $\mathbf{A}$, which under the assumption of being indecomposable is positive and less than 1.

Solving the price system requires to fix one distributional variable and a price invariance equation. For simplicity we assume that the variables depend on r , such that we will use the notation $\mathbf{p}(r)$ and $w(r)$. We then have for $r < R$,

$$\mathbf{p}(r) = w(r)\ell \left((1 + r)^{-1} \mathbf{I} - \mathbf{A} \right)^{-1}. \quad (5)$$

Then note that $\mathbf{p}(0) = w(0)\lambda$, the labor value vector in Equation (2). Moreover, for $r = R$, we have that $w(R) = 0$, and

$$(\mathbf{p}(R)\mathbf{A})(1 + R) = \mathbf{p}(R), \quad (6)$$

where $\mathbf{p}(R)$ is the left eigenvector of $\mathbf{A}$ that is associated with the maximum eigenvalue.

2.3 | Quantity system

For the invariance equations below we require to specify the aggregate output in this economy. Rather than the scale, what is important is how output is distributed among sectors. In particular, the invariance equations depend on either the gross or net product. Define the former as the $n \times 1$ vector, $\mathbf{Q}$, that contains each sector's gross product, and the latter as $\mathbf{q} = (\mathbf{I} - \mathbf{A})\mathbf{Q}$. Labor values satisfy $\lambda \mathbf{q} = \ell(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{I} - \mathbf{A})\mathbf{Q} = \ell \mathbf{Q}$.

⁴As noted by an anonymous referee, according to Marx's (1993) definitions in *Capital Volume 1* (Penguin ed.), *surplus value* (p. 251, 320), *constant and variable capital* (pp. 317, 320) and *profit* (understood as a subset of surplus value, pp. 328–29) are *price* magnitudes, while *necessary labor and surplus labor* (p. 325) are *value* magnitudes. Correspondingly, *the rate of surplus value* is a ratio of price terms, surplus value and variable capital (324, 326), while the *degree of exploitation* is a ratio of value terms (which corresponds to σ here), surplus labor and necessary labor (326–7, 668–69). We follow the framework of Morishima (1973) and Pasinetti (1977) where surplus value is defined as a value magnitude. All the categories defined in this section are in value terms.

Condition 0. We adopt the normalization $\lambda \mathbf{q} = \lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = L = 1$ which applies to both the value system of Section 2.1 and the price system of Section 2.2. Since in both systems labor is the only non-produced input, this imposes an exogenous constraint on the production level.

The profit rate, as measured in values, is

$$r^\lambda = \frac{\alpha}{\lambda \mathbf{A}\mathbf{Q} + (1 - \alpha)} = \frac{\alpha/(1 - \alpha)}{c/v + 1} = \frac{\sigma}{c/v + 1}, \quad (7)$$

where $c/v = \lambda \mathbf{A}\mathbf{Q}/(1 - \alpha)$ is the value composition of capital. Note that if we have a constant appropriation rate, α , the profit rate in values depends on the ratio of $\lambda \mathbf{A}\mathbf{Q}$ to $\ell \mathbf{Q} = 1$. This differs from the price system where the profit rate does not depend on the production distribution and allocation of labor across sectors.

Using Equation (5), $p(r) = w(r)\ell((1 + r)^{-1}\mathbf{I} - \mathbf{A})^{-1}$, for all $r < R$, we obtain

$$p(r)\mathbf{Q} = w(r)\ell((1 + r)^{-1}\mathbf{I} - \mathbf{A})^{-1}\mathbf{Q} = w(r)\varphi(r), \quad (8)$$

where for further reference we define the function

$$\varphi(r) = \ell((1 + r)^{-1}\mathbf{I} - \mathbf{A})^{-1}\mathbf{Q}. \quad (9)$$

3 | VALUE-PRICE SYSTEM SOLUTIONS

Now consider a joint system, where both values and prices are considered. Note that in this system there are $2n + 3$ unknowns: n labor values, n prices, α , the appropriation rate defined above, the profit rate r , and wage w . The main feature of this paper is to produce a typology of transformation procedures based on the additional invariance equations needed to obtain an analytical solution.

Note that the commodities' values λ can be obtained independently using Equation (2). Moreover, as it is well known in the Sraffian model, if a distributive variable is fixed the other variable and the rest of the price-system variables can be obtained. The latter, however, requires an additional invariance condition. As a result, we only require two additional equations.

We will consider models that will be based on aggregate production invariance conditions, A1 or A2, and capital invariant conditions, C1, C2 or C3.⁵

For the former we have:

Condition A1. Aggregate gross product is invariant in prices and values, $\lambda \mathbf{Q} = p\mathbf{Q}$.

Condition A2. Aggregate net product is invariant in prices and values, $\lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = p(\mathbf{I} - \mathbf{A})\mathbf{Q}$.

For the latter:

Condition C1. Aggregate capital in value equals aggregate capital in prices, $\lambda \mathbf{A}\mathbf{Q} + (1 - \alpha) = p\mathbf{A}\mathbf{Q} + w$.

Condition C2. Aggregate variable capital in value equals aggregate variable capital in prices,

$$(1 - \alpha) = w.$$

⁵These are different combinations of Seton (1957) invariance conditions and of Axioms 3 (Aggregate Proportionalities) and 4 (Aggregate Equalities) in Mohun and Veneziani (2017, p. 11).

Condition C3. *Aggregate constant capital in value equals aggregate constant capital in prices,*

$$\lambda \mathbf{A}\mathbf{Q} = \mathbf{p}\mathbf{A}\mathbf{Q}.$$

There is, of course, a well-known special case where all conditions are satisfied if $\lambda \propto \mathbf{p}$, that is, labor values are proportional to prices of production. This particular case is achieved when the value composition is the same across sectors. We will not assume this special case.

Although we will use those 5 conditions above, there is an additional condition that is traditionally assumed to be a key invariance equation in the transformation problem, that aggregate surplus value equals aggregate profits. This could be written as

Condition A3. *Aggregate surplus is invariant in prices and values, $\alpha = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{Q} - \mathbf{w}$.*

Note that A3 is redundant with any other condition. For instance, if we assume A1 and A3, then $\alpha = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{Q} - \mathbf{w} = \lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} + (\lambda - \mathbf{p})\mathbf{A}\mathbf{Q} - \mathbf{w} = 1 - (\lambda - \mathbf{p})\mathbf{A}\mathbf{Q} - \mathbf{w}$ a condition that could be achieved by A1 and C1. If we assume A2 and A3, C2 can be trivially obtained. If we assume A3 and C1, then this implies A1, and so on. As a result we will consider A3 as a result to be checked for any model.

If A1 is assumed then, using Equation (8), we obtain

$$w(r) = \lambda \mathbf{Q}(\varphi(r))^{-1},$$

$$\mathbf{p}(r)\mathbf{A}\mathbf{Q} = \lambda \mathbf{Q} \frac{1 - (1+r)(\varphi(r))^{-1}}{1+r},$$

$$\text{Agg. profits} = \lambda \mathbf{Q} \left(1 - \frac{1 - (1+r)(\varphi(r))^{-1}}{1+r} - (\varphi(r))^{-1} \right) = \lambda \mathbf{Q} \frac{r}{1+r},$$

$$\text{Agg. capital} = \lambda \mathbf{Q} \left(\frac{1 - (1+r)(\varphi(r))^{-1}}{1+r} + (\varphi(r))^{-1} \right) = \lambda \mathbf{Q} \frac{1}{1+r}.$$

If A2 is assumed then also from Equation (8),

$$\mathbf{p}(r)\mathbf{Q} = 1 + \mathbf{p}(r)\mathbf{A}\mathbf{Q} = w(r)\varphi(r),$$

$$(\mathbf{p}(r)\mathbf{A}\mathbf{Q} + w(r))r = 1 - w(r).$$

From the former $\mathbf{p}(r)\mathbf{A}\mathbf{Q} = w(r)\varphi(r) - 1$, while from the second, $\mathbf{p}(r)\mathbf{A}\mathbf{Q} = (1 - w(r))/r - w(r) = \frac{1 - w(r) - r w(r)}{r}$, such that $r w(r)\varphi(r) - r = 1 - w(r) - r w(r)$.⁶ Therefore,

$$w(r) = \frac{1+r}{1+r+r\varphi(r)},$$

$$\mathbf{p}(r)\mathbf{A}\mathbf{Q} = \frac{1+r}{1+r+r\varphi(r)}\varphi(r) - 1 = \frac{(1+r)\varphi(r) - 1 - r - r\varphi(r)}{1+r+r\varphi(r)} = \frac{\varphi(r) - 1 - r}{1+r+r\varphi(r)},$$

$$\text{Agg. profits} = 1 - w(r) = \frac{1+r+r\varphi(r) - (1+r)}{1+r+r\varphi(r)} = \frac{r\varphi(r)}{1+r+r\varphi(r)},$$

⁶Another way of seeing it is using these two equations: Then, $\mathbf{p}(r)\mathbf{A}\mathbf{Q} = w(r) \left(\frac{\varphi(r)}{1+r} - 1 \right) = w(r)\varphi(r) - 1$, which implies that $w(r) \left(\frac{\varphi(r)}{1+r} - 1 - \varphi(r) \right) = w(r) \left(\frac{\varphi(r) - 1 - r - r\varphi(r)}{1+r} \right) = -1$.

$$\text{Agg. capital} = w(r)\varphi(r) - 1 + w(r) = \varphi(r) \frac{1+r}{1+r+r\varphi(r)} - \frac{r\varphi(r)}{1+r+r\varphi(r)} = \frac{\varphi(r)}{1+r+r\varphi(r)}.$$

Note that the two models, A1 and A2, imply different wage rates, and therefore they produce an overdetermined system unless in the very special case of the Standard Commodity (see next section).

4 | DIFFERENT TRANSFORMATION PROCEDURES

4.1 | A1 and C1: The total capital invariant condition

The invariance conditions for this model can be summarized as

$$\lambda \mathbf{A} \mathbf{Q} + (1 - \alpha) = p \mathbf{A} \mathbf{Q} + w,$$

$$\lambda \mathbf{Q} = p \mathbf{Q},$$

$$\lambda (\mathbf{I} - \mathbf{A}) \mathbf{Q} = 1.$$

The gross aggregate condition together with the total capital invariant conditions implies that

$$p(r) \mathbf{A} \mathbf{Q} + w(r) = \lambda \mathbf{A} \mathbf{Q} + (1 - \alpha(r)) = \frac{p(r) \mathbf{Q}}{1+r} = \frac{\lambda \mathbf{Q}}{1+r},$$

then,

$$(\lambda \mathbf{A} \mathbf{Q} + (1 - \alpha(r)))(1+r) = \lambda \mathbf{Q} = \lambda \mathbf{A} \mathbf{Q} + 1.$$

From these equations we obtain the total variable capital measured in values (that is equal to the necessary labor),

$$1 - \alpha(r) = \frac{\lambda \mathbf{Q}}{1+r} - \lambda \mathbf{A} \mathbf{Q} = \frac{1 - r \lambda \mathbf{A} \mathbf{Q}}{1+r},$$

and aggregate surplus value (and the appropriation rate),

$$\alpha(r) = 1 + \lambda \mathbf{A} \mathbf{Q} - \frac{\lambda \mathbf{Q}}{1+r} = \frac{r \lambda \mathbf{Q}}{1+r} = \frac{r(1 + \lambda \mathbf{A} \mathbf{Q})}{1+r}.$$

A simple result from this model is that

$$\text{Agg. surplus} = \alpha(r) = \frac{r \lambda \mathbf{Q}}{1+r} = \frac{r p(r) \mathbf{Q}}{1+r} = \text{Agg. profits}$$

Therefore, as noted in Montes-Rojas (2017), $r = r^\lambda$, which is the profit rate invariant condition in Loranger (2004). Note, however, that this is achieved without imposing a uniform value composition across sectors.

The appropriation rate α is a non-negative monotonic increasing function of r , $\alpha(r)$, with $\alpha(0) = 0$ and $\alpha(R) = \frac{R \lambda \mathbf{Q}}{1+R}$, a value that is achieved by the maximum rate of profits. Note that $\alpha \geq r$ depending on whether $\lambda \mathbf{A} \mathbf{Q} \geq r$. This function also satisfies that $0 \leq \alpha(r) \leq 1$.

Lemma 1. Assume conditions 0, A1 and C1, and that $\mathbf{A}$ is a non-negative indecomposable matrix. Then, $\alpha(r) \leq 1$.

Proof Suppose that $\alpha(R) > 1$. From the price equations we have $(\lambda \mathbf{A}\mathbf{Q} + (1 - \alpha(R))(1 + R) = \lambda \mathbf{Q} = \mathbf{p}(R)\mathbf{Q}$ and $\mathbf{p}(R)\mathbf{A}\mathbf{Q}(1 + R) = \lambda \mathbf{Q} = \mathbf{p}(R)\mathbf{Q}$. This then implies that $\lambda \mathbf{A}\mathbf{Q} \geq \mathbf{p}(R)\mathbf{A}\mathbf{Q}$. Note however that $\lambda = \mathbf{p}(0)$, that is, the price vector that would result when the profit rate is 0. Standard results in the Sraffian model determines that $\mathbf{p}(r)\boldsymbol{\theta}$ is an increasing function in r for any non-negative dimensional vector $\boldsymbol{\theta}$, except for the invariant price equation, which in this case is $\lambda \mathbf{Q} = \mathbf{p}(r)\mathbf{Q}$. Thus this is a contradiction. QED

This determines that this model can obtain the appropriation rate $\alpha(r)$, and then we can define the necessary labor, $1 - \alpha(r)$, which is a non-negative number less or equal to 1, and the surplus rate, $\sigma(r) = \alpha(r)/(1 - \alpha(r))$, which satisfies that $\sigma(r)$ is monotonically increasing in r and that it satisfies $\sigma(0) = 0$ and $\sigma(R) = \alpha(R)/(1 - \alpha(R))$. Moreover, we have that $w(r) = \lambda \mathbf{A}\mathbf{Q} - \mathbf{p}(r)\mathbf{A}\mathbf{Q} + (1 - \alpha(r)) \leq 1 - \alpha(r)$. Note that in this case, wages are lower than the corresponding labor values. As a result we could have that $w(R) = 0 < 1 - \alpha(R)$. In other words, the necessary labor is positive even when wages are zero, which is a negative result.⁷

4.2 | Conditions A2 and C1: The total capital invariant condition with net product

The net aggregate condition together with the total capital invariant conditions implies that

$$\lambda \mathbf{A}\mathbf{Q} + (1 - \alpha) = \mathbf{p}\mathbf{A}\mathbf{Q} + w,$$

$$\lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{Q},$$

$$\lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = 1.$$

Now, expressing everything as a function of the profit rate, $\alpha(r) = 1 + \lambda \mathbf{A}\mathbf{Q} - \mathbf{p}(r)\mathbf{A}\mathbf{Q} - w(r)$. Since $w(r) = \frac{1+r}{1+r+r\varphi(r)}$ and $\mathbf{p}(r)\mathbf{A}\mathbf{Q} = \frac{\varphi(r)-1-r}{1+r+r\varphi(r)}$, then

$$\alpha(r) = 1 + \lambda \mathbf{A}\mathbf{Q} - \frac{\varphi(r)}{1+r+r\varphi(r)},$$

$$1 - \alpha(r) = \frac{\varphi(r)}{1+r+r\varphi(r)} - \lambda \mathbf{A}\mathbf{Q}.$$

However, this model does not satisfy the equality in the profit rates, because

$$r = \frac{\mathbf{p}(r)(\mathbf{I} - \mathbf{A})\mathbf{Q} - w(r)}{\mathbf{p}(r)\mathbf{A}\mathbf{Q} + w(r)} = \frac{1 - w(r)}{\mathbf{p}(r)\mathbf{A}\mathbf{Q} + w(r)},$$

$$r^\lambda(r) = \frac{\alpha(r)}{\lambda \mathbf{A}\mathbf{Q} + (1 - \alpha(r))} = \frac{\lambda \mathbf{A}\mathbf{Q} - \mathbf{p}(r)\mathbf{A}\mathbf{Q} + 1 - w(r)}{\mathbf{p}(r)\mathbf{A}\mathbf{Q} + w(r)} = \frac{(\lambda - \mathbf{p}(r))\mathbf{A}\mathbf{Q} + 1 - w(r)}{\mathbf{p}(r)\mathbf{A}\mathbf{Q} + w(r)}.$$

⁷Moreover, there may exist an interval $r \in (R, R']$, such that $\alpha(r) \geq 0$, $\alpha(R') = 0$, and $w(r) < 0$, positive exploitation with negative wages. This, however, should be discarded if prices are to be non-negative and for this $r \leq R$.

This requires that $(\lambda - p(r))\mathbf{A}\mathbf{Q} = 0$, or that the constant capital is valued in the same way in both the labor value and price system. For this case, $r^\lambda(r) \leq r$ and $r^\lambda(R) = \frac{1 + \lambda\mathbf{A}\mathbf{Q} - 1/R}{1/R} = R(1 + \lambda\mathbf{A}\mathbf{Q}) - 1$.⁸

Moreover, as in the previous case, $w(r) \leq 1 - \alpha(r)$ given that $1 - \alpha(r) = w(r) + (p(r) - \lambda)\mathbf{A}\mathbf{Q}$. Note that $w(0) = 1 - \alpha(0)$ and $1 - \alpha(R) = \frac{1}{R} - \lambda\mathbf{A}\mathbf{Q} \geq 0$.

As noted by an anonymous referee this model may violate FMT1. Consider the case of $\alpha = 0$ (no exploitation). Then define the following function

$$g(r) = p(r)\mathbf{A}\mathbf{Q} + w(r) - \lambda\mathbf{Q} = w(r) \left[1 + \ell \left((1+r)^{-1} \mathbf{I} - \mathbf{A} \right)^{-1} \mathbf{A}\mathbf{Q} \right] - \lambda\mathbf{Q},$$

that follows from the invariance conditions and Equation (5), $p(r) = w(r)\ell \left((1+r)^{-1} \mathbf{I} - \mathbf{A} \right)^{-1}$. For this case $g(0) = 0$, which means that zero profits are compatible with no exploitation. However, given that $g(r)$ is a nonlinear function it could have more than one zero, that is, $g(r) = 0$ for $0 < r < R$. Consider the following example,

$$\mathbf{A} = \begin{bmatrix} 0.7 & 0.1 & 0.7 \\ 0.01 & 0.3 & 0.01 \\ 0.06 & 0.1 & 0.1 \end{bmatrix}, \ell = \begin{bmatrix} 0.2 & 0.5 & 0.3 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}',$$

for which we obtain that $g(r = 0) = 0 = g(r \approx 0.16)$ and $R = 0.303$.

4.3 | Conditions A2 and C2: The New Interpretation

$$\lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = p(\mathbf{I} - \mathbf{A})\mathbf{Q},$$

$$(1 - \alpha) = w,$$

$$\lambda(\mathbf{I} - \mathbf{A})\mathbf{Q} = 1.$$

The ‘New Interpretation’ (NI) uses the aggregate net product, Condition A2, rather than gross product because it interprets capitalist exploitation as appropriation of value added (see Rieu, 2006).

Note that Condition A2 implies that $p\mathbf{q} = \lambda\mathbf{q} = 1$. This result then produces $rp(r)\mathbf{A}\mathbf{Q} + (1+r)w(r) = 1$ such that

$$r = \frac{1 - w(r)}{p(r)\mathbf{A}\mathbf{Q} + w(r)},$$

$$r^\lambda(r) = \frac{\alpha(r)}{\lambda\mathbf{A}\mathbf{Q} + (1 - \alpha(r))} = \frac{1 - w(r)}{\lambda\mathbf{A}\mathbf{Q} + w(r)}.$$

In this case, aggregate surplus equals aggregate profits because $\alpha(r) = 1 - w(r)$. Note that $r \neq r^\lambda$ unless $(\lambda - p(r))\mathbf{A}\mathbf{Q} = 0$, which is not satisfied in general. Moreover, $r^\lambda(r) \geq r$ because $p(r)\mathbf{A}\mathbf{Q} \geq \lambda\mathbf{A}\mathbf{Q}$.

⁸Note that $p(R)\mathbf{A}\mathbf{Q}(1 + R) - p(R)\mathbf{A}\mathbf{Q} = p(R)\mathbf{Q} - p(R)\mathbf{A}\mathbf{Q} = 1$, then $p(R)\mathbf{A}\mathbf{Q} = \frac{1}{R}$.

4.4 | Conditions A1 and C2: The New Interpretation with aggregate gross output

Consider now a variation of the former NI solution but using aggregate gross product (Mohun, 2004, discuss this extension and compares to standard NI):

$$\lambda Q = pQ,$$

$$(1 - \alpha) = w,$$

$$\lambda(I - A)Q = 1.$$

This result implies that $rp(r)AQ + (1 + r)w(r) = p(r)Q - p(r)AQ = \lambda Q - p(r)AQ$, then $p(r)AQ = (1 + r)^{-1}\lambda Q - w(r) = (1 + r)^{-1}\lambda Q - \lambda Q(\varphi(r))^{-1}$

$$r = \frac{p(r)Q - p(r)AQ - w(r)}{p(r)AQ + w(r)} = \frac{\lambda Q - p(r)AQ - \lambda Q(\varphi(r))^{-1}}{p(r)AQ + w(r)},$$

$$r^\lambda(r) = \frac{\alpha(r)}{\lambda AQ + (1 - \alpha(r))} = \frac{\lambda Q - \lambda AQ - \lambda Q(\varphi(r))^{-1}}{\lambda AQ + \lambda Q(\varphi(r))^{-1}}.$$

In this case, aggregate surplus equals aggregate profits because $\alpha(r) = 1 - w(r)$. Note that $r \neq r^\lambda$ unless $(\lambda - p(r))AQ = 0$, which is not satisfied in general. Moreover, $r^\lambda(r) \geq r$ because $p(r)AQ \geq \lambda AQ$. Moreover, $r^\lambda(0) = 0$.

4.5 | Conditions (C1 and C2) or (C1 and C3) or (C2 and C3): The capital invariant condition

$$(1 - \alpha) = w \text{ and/or } \lambda AQ = pAQ \text{ and/or } \lambda AQ + (1 - \alpha) = pAQ + w,$$

$$\lambda(I - A)Q = 1.$$

Montes-Rojas (2017) considers the case of two invariant conditions based on both constant and variable capital, together.

In this case, the denominator of both r and r^λ coincide. However, they differ on the numerator, being $p(r)(I - A)Q - w(r)$ for prices and $\alpha(r) = 1 - w(r)$ for values.

Consider the following aggregate profit equation, $(p(r)AQ + w(r))r = p(r)Q - p(r)AQ - w(r)$, then, $(\lambda AQ + w(r))(1 + r) = (p(r)AQ + w(r))(1 + r) = p(r)Q$. Now using Equation (9), $p(r)Q = w(r)\varphi(r)$, and replacing $(\lambda AQ + w(r))(1 + r) = w(r)\varphi(r)$.

Then,

$$1 - \alpha(r) = w(r) = \frac{\lambda AQ(1 + r)}{\varphi(r) - 1 - r},$$

$$r = \frac{w(r)\varphi(r) - \lambda AQ - w(r)}{\lambda AQ + w(r)},$$

$$r^\lambda(r) = \frac{\lambda Q - \lambda AQ - w(r)}{\lambda AQ + w(r)}.$$

For this case, $r^\lambda(r) \geq r$,⁹ where $r^\lambda(0) = 0$ and $r^\lambda(R) = 1/\lambda A Q$.

4.6 | Conditions (A1 and A2), (A1 and C3) or (A2 and C3): Standard Commodity

Consider now the following conditions:

$$\lambda Q = pQ \text{ or } \lambda(I - A)Q = p(I - A)Q,$$

$$\lambda A Q = p A Q,$$

$$\lambda(I - A)Q = 1.$$

A1 and A2 conditions are equivalent because if we consider Condition A1, $pQ = \lambda Q$ and $pA Q = \lambda A Q$ then $\lambda(I - A)Q = p(r)(I - A)Q$, and the same can be obtained by Condition A2, $\lambda(I - A)Q = p(I - A)Q$ and $pA Q = \lambda A Q$ then $pQ = \lambda Q$. Moreover, Conditions A1 and A2 also imply C3.

As a result it generates two value-price systems given by,

$$(\lambda A Q + 1 - \alpha(r^\lambda))(1 + r^\lambda) = \lambda Q,$$

$$(p(r)A Q + w(r))(1 + r) = p(r)Q.$$

Note that these equations generates a mutually consistent system where labor surplus is distributed proportional to total capital. The two equations are independent of each other, and unless we establish a causal link (i.e. observed profits correspond to surplus value), they impose no restrictions on each other. It does however avoid inconsistencies which were observed in the previous solutions. Unfortunately, this model cannot be sustained unless in a very special case. The reason is that if we apply all the invariance conditions and obtain expressions for $w(r)$ then, in general, $\frac{\lambda Q}{\varphi(r)} \neq \frac{1+r}{1+r+r\varphi(r)}$, which are two different expressions for wages that need to be satisfied. The reason is that we cannot, in general, simultaneously maintain gross and net output invariant.

The special case where all the conditions are satisfied is that of the Standard Commodity (as defined in Sraffa, 1960), Q^* such that $A Q^* = \frac{Q^*}{1+R}$. This is the right eigenvector of A associated with the maximum eigenvalue $\frac{1}{1+R}$. This implies that $p(r)A Q^* = \frac{p(r)Q^*}{1+R}$ and in particular that $\lambda A Q^* = \frac{\lambda Q^*}{1+R}$. If we consider Condition A1, $p(r)Q^* = \lambda Q^*$, then $p(r)A Q^* = \lambda A Q^*$ and $\lambda(I - A)Q^* = p(r)(I - A)Q^*$. The same can be obtained by Condition A2, $\lambda(I - A)Q^* = p(I - A)Q^*$, and then this implies that $pQ^* = \lambda Q^*$ and $pA Q^* = \lambda A Q^*$.

These conditions were developed by Eatwell (1975), although for the neo-Ricardian model with a wage bundle, and make him to conclude that by using the Standard Commodity as defined in Sraffa, then this would produce an invariant condition between the price system and the value system, using similar arguments to that of the New Interpretation:

These problems may be overcome by returning to Marx's definition (I) [as the value of "the sum of money v expended upon the labor power," -" in effect as the share of wages in the value of output], making units of standard net product our "money" and measur-

⁹This can be proven by noting that $w(r)\varphi(r) \leq \varphi(r) \leq \lambda Q$.

ing necessary labor time in terms of this “money.” The total amount of labor performed in one production period and the amount of labor embodied in one unit of standard net product are equal. (p. 554)

Araujo (2019) explores the compatibility of the two solutions and reached the same conclusion:

Standard Commodity can be a consistent solution to the transformation problem of labour values into prices of production because it satisfies rigorously the mathematical condition of invariability in relation to income distribution between wages and profits. (p. 615)

Both Eatwell (1975) and Araujo (2019) define their system in terms of the neo-Ricardian model where a fixed consumption bundle is associated with wages. An interesting model is developed by Roberts (2009) where the redistribution of surplus is studied in this framework. As noted above, it generates two independent systems. This is because by using $p(r)(I - A)Q^* = 1$ and $p(r)AQ^* = 1/R$, we obtain

$$w(r) = \frac{(R - r)}{R + Rr}, \quad r = \frac{R(1 - w(r))}{1 + Rw(r)}.$$

Moreover, form $\lambda(I - A)Q^* = 1$, then we obtain $(1 + R)\lambda AQ^* - \lambda AQ^* = 1$ then, $\lambda AQ^* = 1/R$, such that

$$\begin{aligned} \alpha(r^\lambda) &= \frac{r^\lambda(1 + R)}{R(1 + r^\lambda)}, \\ 1 - \alpha(r^\lambda) &= 1 - \frac{r^\lambda(1 + R)}{R(1 + r^\lambda)} = \frac{R(1 + r^\lambda) - r^\lambda(1 + R)}{R(1 + r^\lambda)} = \frac{(R - r^\lambda)}{R(1 + r^\lambda)}, \\ r^\lambda &= \frac{R\alpha(r^\lambda)}{1 + R(1 - \alpha(r^\lambda))}. \end{aligned}$$

Note that the use of the Standard Commodity in fact produces equivalent price and value systems, where both $w(r)$ and $1 - \alpha(r^\lambda)$ represent the shares of the net output paid to workers, as opposed to $1 - w(r)$ and $\alpha(r^\lambda)$ that is appropriated by capitalists. However, there is no relation between the value and price systems unless an additional invariant condition is imposed.

Let $Q^* = C(Q)Q$ where $C(Q)$ is a diagonal matrix that applies the corresponding shares of production that will result in the Standard Commodity. Then, $p(r)AC(Q)Q = \lambda AC(Q)Q = 1/R$. Then we could define the transformation $A^*(Q) = AC(Q)$ where $p(r)A^*(Q)Q = \lambda A^*(Q)Q = 1/R$. Note that $A^*(Q)$ could be interpreted as a different technology that corresponds to the Standard Commodity simple reproduction scheme.

5 | TYPOLOGY OF TRANSFORMATION SOLUTIONS

From the previous analysis it is clear that there are six alternative transformations. The different solutions will be judged on (i) FMT1, how the surplus value rate (or the appropriation rate) relate to the

profit rate, and (ii) on FMT2, the comparison of the money wage and the necessary labor. Tables 1 and 2 summarize the main analytical results of the solutions.

The cases A1-A2, A1-C3 and A2-C3 have an equality of both the profit rate in prices and values, and of wages in prices and necessary labor. The problem here is that we cannot achieve a consistent solution that connects the value and price system, and thus solves for the endogenous surplus rate, unless we have the quantities given by the case of the Standard Commodity. The other cases do not have this problem.

All other models except A2-C1 satisfy the FMT1 in a weak form, implying that exploitation is a necessary condition for profits. As noted above the model A2-C1 could have more than one solution, for which one of them may have that $\alpha = 0$ but the model is solved for $r > 0$. Nevertheless for $\alpha = 0$ there is one solution that has $r = 0$. The relationship also goes in the other way, for all models, a value of $r = 0$ is associated with $\alpha(0) = \sigma(0) = 0$ and $\frac{\partial \sigma(r)}{\partial r} > 0$. This determines that value level categories, that is, either α , σ or r^λ , which are all linked together, can be obtained with the expected relation in terms of the (price level) profit rate. The proposed transformation procedures does not have an a priori causal relationship between prices and values, and as such, this result can be interpreted both ways. First, implying that different distributional configurations (i.e. changes in r) determines value level exploitation (i.e. changes in either α , σ or r^λ). Second, indicating that changes in the price system distributional variables are the result of changes in exploitation at the value level.

The explored solutions differ in terms of the posed FMT2 condition. The solution based on A1-C1 and A2-C1 are rather unsatisfactory because there could be a positive necessary labor, even when money wages are zero, that is, $w(R) = 0 < 1 - \alpha(R)$. The NI model A2-C2 (and its variant A1-C2) and the capital invariant transformations (C1-C2, C1-C3, C2-C3) have by construction that wages are equal to the necessary labor, thus $w(R) = 0 = 1 - \alpha(R)$.

TABLE 1 Summary of transformation solutions: equalities and inequalities

	A1	A2	C1	C2	C3
A1	-	^a $r = r^\lambda$ $w = 1 - \alpha$	$r = r^\lambda$ $w \leq 1 - \alpha$	$r \leq r^\lambda$ $w = 1 - \alpha$	^a $r = r^\lambda$ $w = 1 - \alpha$
A2	-	-	$r \geq r^\lambda$ $w \leq 1 - \alpha$	$r \leq r^\lambda$ $w = 1 - \alpha$	^a $r = r^\lambda$ $w = 1 - \alpha$
C1	-	-	-	^b $r \leq r^\lambda$ $w = 1 - \alpha$	^b $r \leq r^\lambda$ $w = 1 - \alpha$
C2	-	-	-	-	^b $r \leq r^\lambda$ $w = 1 - \alpha$
C3	-	-	-	-	-

^aA1-A2, A1-C3, A2-C3 Standard Commodity. A1-C1 total capital invariant solution. A2-C1 total capital invariant solution with net product. A2-C2 New Interpretation. A1-C2 New Interpretation with gross product.

^bC1-C2, C1-C3, C2-C3 capital invariant solution.

TABLE 2 Summary of transformation solutions, analytical solutions

	$w(r)$	$1 - \alpha(r)$	$\alpha(r)$	$r^\lambda(r)$	$\sigma(r)$
A1-C1	$\lambda Q(\varphi(r))^{-1}$	$\frac{1 - r\lambda Q}{1 + r}$	$\frac{r(1 + \lambda Q)}{1 + r}$	r	$\frac{r(1 + \lambda Q)}{1 - r\lambda Q}$
A2-C1	$\frac{1 + r}{1 + r + r\varphi(r)}$	$1 + \lambda Q - \frac{\varphi(r)}{1 + r + r\varphi(r)}$	$\frac{\varphi(r)}{1 + r + r\varphi(r)} - \frac{\lambda Q}{1 + r}$	$\frac{1 - u(r)}{1 + r}$	$\frac{\varphi(r) - \lambda Q(1 + r + r\varphi(r))}{(1 + \lambda Q)(1 + r + r\varphi(r)) - \varphi(r)}$
A2-C2	$\frac{1 + r}{1 + r + r\varphi(r)}$	$\frac{1 + r}{1 + r + r\varphi(r)}$	$\frac{r\varphi(r)}{1 + r + r\varphi(r)}$	$\frac{1 - u(r)}{\lambda Q + u(r)}$	$\frac{r\varphi(r)}{1 + r}$
A1-C2	$\lambda Q(\varphi(r))^{-1}$	$\lambda Q(\varphi(r))^{-1}$	$1 - \lambda Q(\varphi(r))^{-1}$	$\frac{\lambda Q - \lambda Q - \lambda Q(\varphi(r))^{-1}}{\lambda Q + \lambda Q(\varphi(r))^{-1}}$	$(\lambda Q)^{-1} \varphi(r) - 1$
C1-C2	$\frac{\lambda Q(1 + r)}{\varphi(r)(1 - r)}$	$\frac{\lambda Q(1 + r)}{\varphi(r)(1 - r)}$	$\frac{\varphi(r) - 1 - r - \lambda Q(1 + r)}{\varphi(r)(1 - r)}$	$\frac{\lambda Q - \lambda Q - u(r)}{\lambda Q}$	$\frac{\varphi(r) - 1 - r - \lambda Q(1 + r)}{\lambda Q(1 + r)}$
C2-C3					
C1-C3					

Note: $\ell = (r)\phi^{-1}I - (r + 1)Q$.

6 | NUMERICAL EXAMPLE

Consider the example in Sraffa (1960, Section 25) with 3 commodities that correspond to basic industries that produce iron, coal and wheat in the following way:

$$90 \text{ t.iron} \oplus 120 \text{ t.coal} \oplus 60 \text{ qr. wheat} \oplus \frac{3}{16} \text{ labour} \rightarrow 180 \text{ t.iron},$$

$$50 \text{ t.iron} \oplus 125 \text{ t.coal} \oplus 60 \text{ qr. wheat} \oplus \frac{5}{16} \text{ labour} \rightarrow 450 \text{ t.coal},$$

$$40 \text{ t.iron} \oplus 40 \text{ t.coal} \oplus 200 \text{ qr. wheat} \oplus \frac{8}{16} \text{ labour} \rightarrow 480 \text{ t.wheat}.$$

Since iron is produced in quantity just sufficient for replacement, then the net product consists

of only coal (165 t.) and wheat (70 qr.). This model can be represented as $A = \begin{pmatrix} \frac{120}{180} & \frac{120}{180} & \frac{60}{180} \\ \frac{50}{450} & \frac{125}{450} & \frac{60}{450} \\ \frac{40}{480} & \frac{40}{480} & \frac{200}{480} \end{pmatrix}$,

$\ell = \left(\frac{3}{16}, \frac{5}{16}, \frac{8}{16} \right)$ and $Q' = (1, 1, 1)$. In this case, the maximum profit rate is $R = 0.20$, thus the model guarantees non-negative prices for $r \in [0, R]$.

Figure 1 presents the estimated $\alpha(r)$ exploitation rate functions for all the studied cases. The main differences are observed for A1–C1 and A2–C2 that are the ones that are below the rest, with $\alpha(R) < 1$, but for the others $\alpha(R) = 1$. Figure 2 presents $1 - \alpha(r)$, the necessary labor, while Figure 3 reports the implied surplus rate. Finally, Figure 4 plots the profit rates in values and prices. For the latter, the only solution with the same rate in both prices and values is A1–C1.

Note that for this A1–C1, for $R = 0.20$, we have $\sigma(0.20) = 20.9$ and $\alpha(0.20) = 0.954$, while for the A2–C1, $\sigma(0.20) = 2.648$ and $\alpha(0.20) = 0.726$. This determines that for the former case the

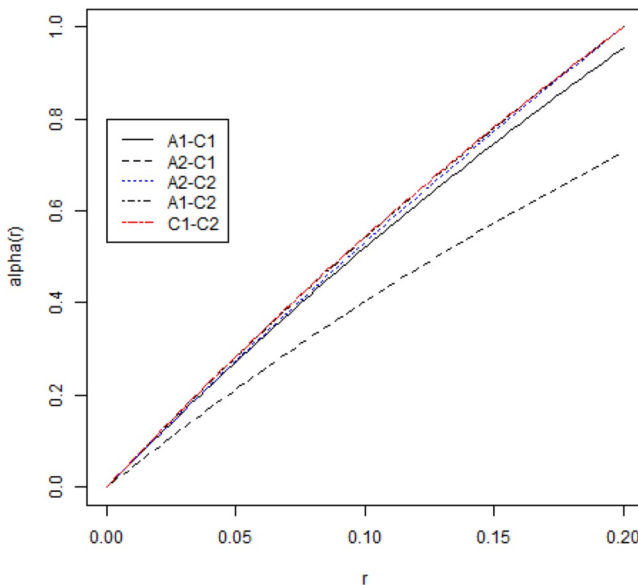


FIGURE 1 $\alpha(r)$ appropriation rate functions [Colour figure can be viewed at wileyonlinelibrary.com]

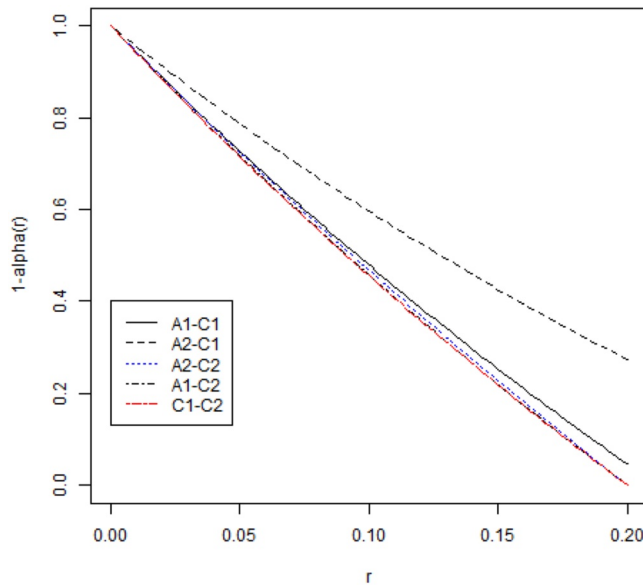


FIGURE 2 $1 - \alpha(r)$ necessary labor functions [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/meca.12406)]

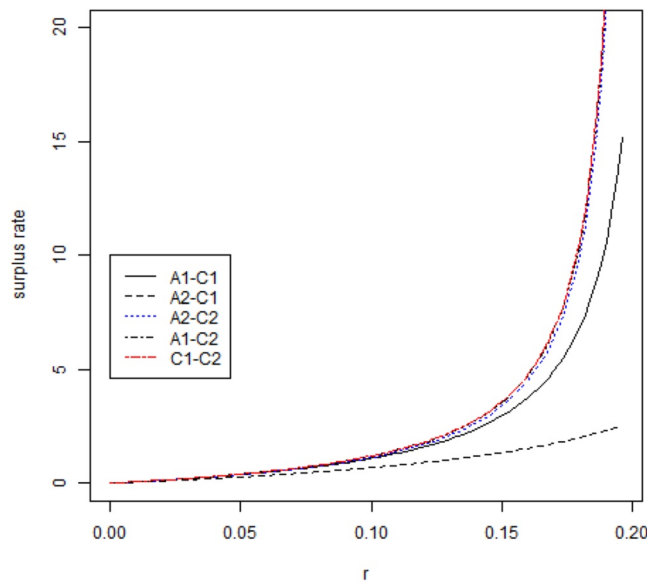


FIGURE 3 $\sigma(r)$ surplus rate functions [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/meca.12406)]

maximum exploitation that is compatible with the price system is 95.4% of total labor, while for the latter 72.6%. These numerical results illustrate the case of positive necessary labor when wages are zero (i.e. $1 - \alpha(0.20) > 0$).

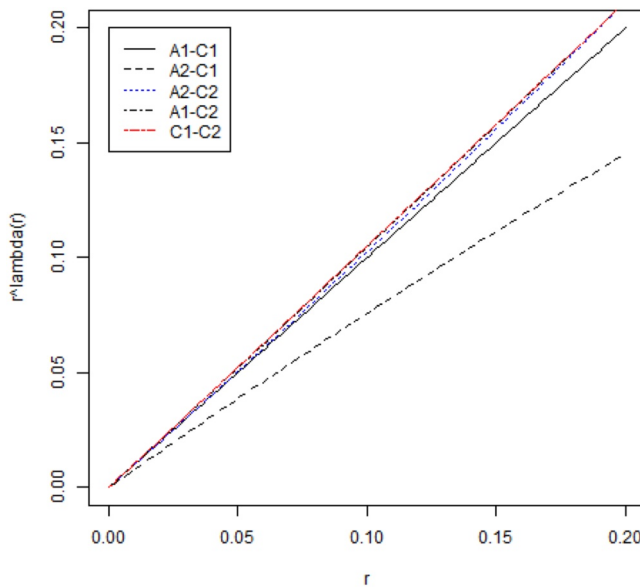


FIGURE 4 $r^\lambda(r)$ profit rate in values functions [Colour figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com/doi/10.1111/meca.12406)]

7 | CONCLUSION AND DISCUSSION

This paper proposes a unified framework to evaluate different transformation solutions. The proposed procedure allows for a simple computation of key variables in the economic system, which are of interest for economists with interest in measures of surplus, either within the Ricardian, Marxian or Sraffian perspective, what Garegnani (1984) calls the surplus approach. The proposed model solves for the implied appropriation rate, from which the surplus rate and the necessary labor can be obtained, to satisfy different value-price invariance equations. Thus it proposes to get this value-level measure endogenously.

The main purpose of this analysis is that distributive parameters that are specific to the prices of production system can be recovered from a joint solution. Moreover, the movements along the wage-profit frontier that correspond to different distributive configurations in an economy (as in the Sraffian framework) can be traced to the labor value analysis. As noted by an anonymous referee, the proposed framework determines that the different procedures can be either interpreted as α -determined, r -determined or w -determined, depending on which parameter index is used to study distributional features.

The different solutions are judged on whether they satisfy two conditions, defined as FMT1 and FMT2, which correspond to the relation between profit and surplus value rates, and total variable capital and money wages, respectively. An analysis of the solutions determines that the NI and the capital invariant solutions are the most appropriate. In turn, this paper proposes that if using any of these two alternatives, the Marxian value system can be incorporated to a Sraffian price system if the intention is to use value level categories to explain the distribution of income.

Petri (2015) criticizes recent transformation procedures since the NI, many of which are all special cases of our proposed framework. This author argues that, in the proposed solutions, the labor theory of value loses the role of instrument for the determination of the rate of profit and relative prices, and is redefined as the reinterpretation of independently determined prices as ‘representing’ quantities of labor, on the basis of the postulate that only labor produces exchange value. While we agree with this

postulate we claim that social relations can always be interpreted in different schemes. The exploitation and surplus value rates are good proxies of the relative power of labor and capital. The workers' self-consciousness of their importance for the reproduction of the capitalist system can exert direct influence on the exploitation rate, which in turn, affects price-level distributive variables.

REFERENCES

- Araujo, F.A. (2019). Sraffa and the labour theory of value: A note. *Brazilian Journal of Political Economy*, 39(4), 614–637. <https://doi.org/10.1590/0101-31572019-2859>
- Duménil, G. (1983). Beyond the transformation riddle: A labor theory of value. *Science and Society*, 47(4), 427–450.
- Eatwell, J. (1975). Mr. Sraffa's standard commodity and the rate of exploitation. *Quarterly Journal of Economics*, 89(4), 543–555. <https://doi.org/10.2307/1884691>
- Fine, B., Lapavistas, C., & Saad-Filho, A. (2004). Transforming the transformation problem: Why the 'New Interpretation' is a wrong turning. *Review of Radical Political Economics*, 36(1), 3–19. <https://doi.org/10.1177/0486613403261091>
- Foley, D. K. (1982). The value of money, the value of labor power, and the Marxian transformation problem. *Review of Radical Political Economics*, 14(2), 37–47. <https://doi.org/10.1177/048661348201400204>
- Foley, D. K. (1986). *Understanding capital. Marx's economic theory*. Harvard University Press.
- Foley, D. K. (2000). Recent developments in the labor theory of value. *Review of Radical Political Economics*, 32(1), 1–39. [https://doi.org/10.1016/S0486-6134\(00\)88759-8](https://doi.org/10.1016/S0486-6134(00)88759-8)
- Garegnani, P. (1984). Value and distribution in the classical economists and Marx. *Oxford Economic Papers*, 36(2), 291–325. <https://doi.org/10.1093/oxfordjournals.oep.a041640>
- Glick, M., & Ehrbar, H. (1987). The transformation problem: An obituary. *Australian Economic Papers*, 26(49), 294–317. <https://doi.org/10.1111/j.1467-8454.1987.tb00510.x>
- Kurz, H.D., & Salvadori, N. (1995). *Theory of production. A long-period analysis*. Cambridge University Press.
- Loranger, J.-G. (2004). A profit-rate invariant solution to the Marxian transformation problem. *Capital&Class*, 82(1), 23–58. <https://doi.org/10.1177/030981680408200104>
- Marx, K. (1993). *Capital: A critique of political economy* (volumes 1, 2 and 3). Penguin Classics.
- Mohun, S. (1994). A re(in)statement of labour theory of value. *Cambridge Journal of Economics*, 18(4), 391–412. <https://doi.org/10.1093/oxfordjournals.cje.a035282>
- Mohun, S. (2004). The labour theory of value as foundation for empirical investigations. *Metroeconomica*, 55(1), 65–95. <https://doi.org/10.1111/j.0026-1386.2004.00183.x>
- Mohun, S., & Veneziani, R. (2017). Value, price, and exploitation: The logic of the transformation problem. *Journal of Economic Surveys*, 31(5), 1387–1420. <https://doi.org/10.1111/joes.12223>
- Montes-Rojas, G. (2017). A capital invariant solution to the Marxian transformation problem. *Review of Radical Political Economics*, 49(1), 114–124. <https://doi.org/10.1177/0486613415616216>
- Morishima, M. (1973). *Marx's economics: A dual theory of value and growth*. Cambridge University Press.
- Moseley, F. (2000). The 'New Solution' to the transformation problem: A sympathetic critique. *Review of Radical Political Economics*, 32(2), 282–316. <https://doi.org/10.1177/048661340003200205>
- Pasinetti, L. (1977). *Lectures on the theory of production*. Columbia University Press.
- Patniak, P. (1989). Marx's transformation problem and Ricardo's invariant measure. *Cambridge Journal of Economics*, 13(4), 555–562. <https://doi.org/10.1093/oxfordjournals.cje.a035111>
- Petri, F. (2015). On some modern reformulations of the labour theory of value. *Contributions to Political Economy*, 34(1), 77–104. <https://doi.org/10.1093/cpe/bzv003>
- Rieu, D.-M. (2006). A reexamination of the quantitative issues in the New Interpretation. *Review of Radical Political Economics*, 38(2), 258–271. <https://doi.org/10.1177/0486613405285429>
- Roberts, B. B. (2009). Ricardo: standard commodity: Marx. *Review of Political Economy*, 21(4), 589–619. <https://doi.org/10.1080/09538250903214875>
- Seton, F. (1957). The transformation problem. *Review of Economic Studies*, 24(3), 149–160. <https://doi.org/10.2307/2296064>
- Sinha, A. (2000). The transformation problem: Is the Standard Commodity a solution? *Review of Radical Political Economics*, 32(2), 265–281. [https://doi.org/10.1016/S0486-6134\(00\)80016-9](https://doi.org/10.1016/S0486-6134(00)80016-9)
- Sraffa, P. (1960). *Production of commodities by means of commodities*. Cambridge University Press.

Steedman, I. (1977). *Marx after Sraffa*. New Left Books.

Wolff, R., Roberts, B., & Callari, A. (1982). Marx's (not Ricardo) 'transformation problem': A radical conceptualization. *History of Political Economy*, 14(4), 564–582. <https://doi.org/10.1215/00182702-14-4-564>

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