

The Chaos Theory and its Application

Feier Su

Xi'an Gaoxin NO.1 high school, Shaanxi, Xi'an 710019, China

*Corresponding author's e-mail: sufeier7@gmail.com

Abstract. Chaos Theory is not only one of the most active branches of the dynamical system, but also one of the most important topics of the nonlinear scientific field. The main purpose of this thesis is to introduce and illustrate the basic concepts, characteristics, and various applications of Chaos Theory. Chaos Theory is widely applied in physics, biology, and economics fields etc., so the thesis will also exhibit interdisciplinary research on Chaos Theory. The conclusion is that the most important refinement of Chaos Theory is its sensitivity to initial values, and subtle changes will affect the final shape and state of the entire structure.

1. Introduction

Chaos theory originated in the second half of the 20th century, and its origin is closely related to weather and meteorology. Scientists have made a dialectical and logical definition of chaos theory, which can be summarized as that the random state generated in a deterministic system is an irregular state on the surface. But in fact it is the movement produced by the system movement according to the non-linear dynamic system.

2. Literature review

2.1. Nonlinear and linear system

Non-linear system means that the output is not proportional to the input, and the frequency may change. The main feature of the linear system is that the output brightness is proportional to the input, and the frequency is unchanged. Non-linearity is the essence of the real world, so with the progress of science, the development of non-linear science must be an inevitable trend. At present, nonlinear science has formed three frontier theories: soliton theory, fractal theory and chaos theory.

2.2. Fractal

Fractal is an important concept of fractal geometry theory created by the famous mathematician Mandelbrot, which means that the system has self-similar properties under different scales. Self-similarity is symmetry across scales, that is, there is another pattern within a pattern. Because the system features have cross-scale repetitiveness, it can produce hidden ordered patterns with structure and rules. Therefore, fractals have two common characteristics:

- A. They are irregular from beginning to the end.
- B. The degree of irregularity is indeed constant on different scales.

2.3. Strange attractor

An attractor is a form in which the system is attracted and always fixed in a certain state. Three different attractors control and limit the motion of objects: point attractors, limit cycle attractors, and strange



attractors. Both the point attractor and the limit cycle attractor play a limiting role. The shape of the system presents static and balanced characteristics, so they are also called convergent attractors. The singular attractor is different from the previous two. The singular attractor is a product reflecting the characteristics of the movement of a chaotic system, and it is also a disordered and steady-state of movement in a chaotic system. It makes the system deviate from the area of the convergent attractor and lead to a different shape. It induces the vitality of the system and turns it into a non-predetermined mode, which is unpredictable.

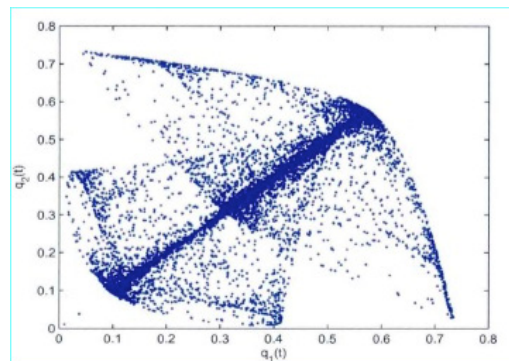


Figure1. The strange attractor [2]

2.4. Bifurcation and bifurcation points

It means that when a certain parameter or a certain group of parameters changes, the type of long-term dynamic motion also changes. This parameter value (or this group of parameter values) is called the bifurcation point. Small changes in the parameters at the bifurcation point will produce different dynamic characteristics, so the system is structurally unstable at the bifurcation point.

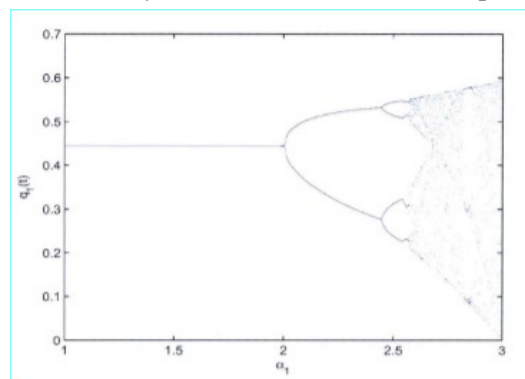


Figure 2. The bifurcation

3. Important characteristics and qualities

3.1. Sensitivity affected by the initial state

In Chaos Theory, sensitivity to initial values is a very distinctive feature of chaos. Very small changes in the initial conditions can also lead to huge differences in the final state. The famous "butterfly effect" can explain this chaotic nature. The "butterfly effect" originally refers to a butterfly flapping its wings as far away as Brazil, South America. This air movement may eventually trigger a tornado in the United States of North America. According to the butterfly effect, the Chaos Theory believes that in a dynamic system, a small change in the initial conditions triggers a huge chain reaction of the entire system. The changes and development of the system are nonlinear and unpredictable. The result of the development is due to the initial conditions. Have sensitive dependencies. Whenever a certain degree of sensitivity to tiny initial conditions occurs, the butterfly effect can prompt the system to adapt to new environments and new conditions to make nonlinear changes

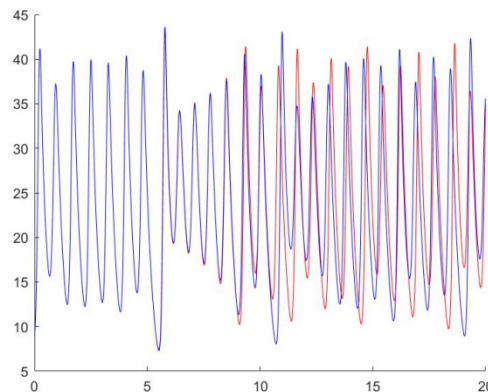


Figure 3 Sensitivity affected by the initial state

3.2. Randomness

This is a reflection of the internal randomness of a completely deterministic system, rather than external randomness, without any additional random factors. The inherent randomness of chaos is that its unpredictability and sensitivity to initial values make it this property. It also shows that chaos is locally unstable.

3.3. Universality

Refers to some common characteristics shown by different systems tending to chaos. It will not change due to differences in systems and system motion equations.

3.4. Ergodicity

Chaotic motion is ergodic in the field of chaotic attraction. That is, the orbit passes through every state point in the chaotic zone in a limited time.

3.5. Boundedness

Chaos is bounded. Its trajectory is always limited to a certain area. This area becomes a chaotic attraction zone. No matter how uncertain the chaotic system is, its trajectory does not go out of this chaotic attraction zone. So on the whole, chaos is stable.

4. Chaos Identification Method

a. Lyapunov exponent: By calculating the Lyapunov exponent, if the maximum Lyapunov exponent of the system is positive, the system is considered to be chaotic.

b. Hausdorff Dimension: By calculating the Hausdorff dimension of the system. If these dimensions are fractal, the system is chaotic.

c. Melnikov method: By calculating the Melnikov function, it can be determined whether the homoclinic or heteroclinic trajectory system can appear in chaos after being disturbed.

d. Power spectrum method: By drawing the power spectrum image, the power spectrum only has discrete spectral lines at the moving frequency and its division and multiplication. If the power spectrum that appears is continuous, the system is chaotic. [2]

5. Interdisciplinary Research on Chaos Theory

5.1. Research on Chaos Theory in Economics

In a chaotic economic system, the system is the result of many interacting individuals constantly adjusting their relationships under unstable conditions, rather than the result of market stability and equilibrium of supply and demand. Each individual is based on its future predictions and other individuals' responses.

To take action, and continue to learn and adapt. From the internal point of view of the system, the movement of individuals is not stable; from the overall point of view, these individuals are confined to the structure, thus forming local randomness and overall stability.

The theoretical research of chaotic economic system mainly includes two types: system method and dynamic method.

For the study of system theory, chaotic economic theory has made major breakthroughs in financial market fluctuations, macroeconomic growth, and microeconomic fragility through the improvement of classical economics, and has deeply portrayed the non-random in economic systems. The unbalanced nature of volatility has unearthed the endogenous structure of the chaotic economic system at different levels and dimensions, refuting the traditional economic theory with exogenous disturbance as the core.

For the study of dynamic methods, the main research is based on the difference equation to study the dynamic characteristics of the attractor around the fixed point. The research frontiers of system periodicity theory are mainly to sort and classify periodic orbits, and analyze the optimization of parameters under different system states.

5.2. Research on Chaos Theory in Language Learning

The sensitive dependence on the initial state in Chaos Theory can provide a reasonable explanation for mother tongue transfer in language learning. Although children will continue to make mistakes in the early stages of learning their mother tongue, they are not disturbed by other languages, so they can use their mother tongue fluently and develop habits in the end. However, when one accepts a new language, the previously mastered language becomes unstable. They are affected by external influences and adapt to the new environment by changing the internal language structure. According to the explanation of Lightbown (1985), after learners have mastered some grammar rules, they encounter new grammar rules. The conflicting grammar rules eventually lead to the reconstruction of the entire system. This is a typical reflection of the sensitivity to the initial state in language learning. [5]

b. The characteristics of "singular attractors" can also be used to study language learning. In language learning, students' personality, learning style, interest, and motivation are like "singular attractors". If these chaotic attractors are fully utilized, they will promote the process of language learning. Take English learning as an example. Because there are many English knowledge points, crisscrossing, the content is not very coherent, and there are many similar knowledge points in the learning process, so in the learning process, it is necessary to cultivate personality and interest, and establish effective learning Style and motivation, constantly compare, classify, summarize and review the knowledge points learned. Only in this way can the learning effect be effectively improved and good academic performance can be achieved.

c. Although chaotic motion is random in the entire time course, prediction is still possible within a certain time range. But it cannot predict its movement in long run. As a nonlinear system, the long-term behavior of the teaching system is also unpredictable. For most students, due to the lack of accurate understanding of the nonlinear characteristics of the teaching system, the learning goals are set regardless of the length of time, and definite learning goals are set. And plan the exam results at the end of the semester. Chaos Theory shows that it is meaningless to set long-term deterministic learning goals and plan exam scores at the end of the semester, because the precise value of the initial state of the system is impossible to obtain, and it is impossible to precisely control the development process of the system. However, short-term or periodic specific goals are conducive to learners to complete learning tasks. From a practical and empirical point of view, the long-term learning process does not need to grasp its details, but the overall trend. Therefore, when formulating learning goals, the most appropriate way is to combine short-term goals with long-term goals

5.3. Research on Chaos Theory in Bioscience

The research of chaos science in biochemistry is more extensive and far-reaching. Life scientists believe that the human body is a complex and highly nonlinear system. If a purely linear method is used to study the complexity of the human body only by adding variables, many difficult problems and information

cannot be completely solved. But now scientists are trying to solve this difficulty through simple nonlinear mathematical models. After the efforts of many life scientists, the application of chaos theory in life sciences has made significant progress, including epidemiology (such as SARS, avian influenza, AIDS, measles, etc.), nervous system, heart rhythm, biochemistry, DNA and protein molecules All of the chaotic forms have improved to varying degrees.[5]

6. Conclusion

The most important refinement of Chaos Theory is its sensitivity to initial values, and subtle changes will affect the final shape and state of the entire structure. The emergence of Chaos Theory has allowed scholars to further study nonlinear science. Its theory can move flexibly in various fields, whether it is economics, philosophy, or education. It has made achievements, and some even broke traditional concepts. At the same time, the deeper meaning and value of chaos are still being explored.

Acknowledgement

First and foremost, I would like to show my deepest gratitude to my teachers and professors in my university, who have provided me with valuable guidance in every stage of the writing of this thesis. Further, I would like to thank all my friends and roommates for their encouragement and support. Without all their enlightening instruction and impressive kindness, I could not have completed my thesis.

References

- [1] Deng Jun. Talking about Nonlinear Science. China Aviation News. 2020.9.15
- [2] Li Yan. Chaos theory and its application in economic systems [D]. Jilin University, 2016.
- [3] He Xiaoxing, Zhao Hua. A review of the application of chaos theory in financial economics and macroeconomics [J]. Financial Research, 2006(07): 166-173.
- [4] Chen Xiaoyi. Chaos Theory and Its Application in Language Research[J]. Modern Chinese, 2018(05): 27-31.
- [5] Zhang Le, Sun Hongwen. Overview of the research on the development of chaos science [J]. China Collective Economy, 2009(13): 110-111.