

Equilibrium in Open Market Models with Nonconstant Elasticities

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Abstract—This paper considers an open market model, where the elasticities are power functions of the price. We obtain sufficient conditions on equilibrium existence for this model and illustrate it with a numerical experiment.

Index Terms—equilibrium, demand, supply, coincidence point, covering mapping, elasticity

I. INTRODUCTION

In modern economics, the notion of equilibrium plays a leading role. Let us describe it generally. Assume that the market consists of two participant groups: producers and consumers. The producers deliver specific goods to the market and sell them. The consumers purchase these goods on the salary they obtain from the producers. The total amount of the goods produced is called the supply. The total amount of the goods needed is called the demand.

Suppose some goods are unsold (i.e., the supply exceeds the demand). Then, the producers lose the income on these goods. If they want to balance it, they can cut the consumers' salaries or raise the prices of goods. One way or another, the consumers cannot afford the amount of goods they purchased in the previous trading cycle. This leads to unsatisfied demand, negatively affecting the market and the region.

On the other hand, if the demand exceeds the supply, some consumers have not received the goods they want or need. This leads to unfavorable situations in the region (it may be massive hunger, lack of building materials, pandemic etc.). This also negatively affects the region.

Combining all these considerations, we look for the situation where all the goods sold and the interest of the consumers is satisfied. This situation is called an equilibrium in the market, and the goods' prices in equilibrium are called equilibrium prices.

In this paper, we apply the theory of covering mappings and coincidence points to an open market model (i.e., the market with import). In this open market model, supply and demand functions are restored by so-called price elasticities. The price elasticity of supply shows the sensitivity of some good's supply to the price of this good or another good. The price elasticity of demand has a similar meaning. Unlike [?] – [5], we consider an open market model where the supply and

demand price elasticities are not constants, but power functions of the price.

II. PROBLEM STATEMENT

Denote $\mathbb{R}_+^n = \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : x_i > 0, i = \overline{1, n}\}$. Let $n \in \mathbb{N}$ goods presented on the market. Their prices are described by a vector $p \in \mathbb{R}_+^n$. This vector is subordinated by the following conditions: there exist vectors $c_1, c_2 \in \mathbb{R}_+^n$, $c_1 = (c_{11}, \dots, c_{1n})$, $c_2 = (c_{21}, \dots, c_{2n})$ such that

$$0 < c_{1i} < c_{2i}, c_{1i} \leq p_i \leq c_{2i}, i = \overline{1, n}.$$

Consider supply and demand functions $D, S : \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$, $D(p) = (D_1(p), \dots, D_n(p))$, $S(p) = (S_1(p), \dots, S_n(p))$. Denote $P = [c_{11}, c_{21}] \times \dots \times [c_{1n}, c_{2n}]$. Suppose we know a vector $p^* \in P$ consisting of the known prices. We also know the supply and the demand for these prices:

$$D(p^*) = D^*, \quad (1)$$

$$S(p^*) = S^*. \quad (2)$$

Also, suppose we know the import vector $a \in \mathbb{R}^n$, $a = (a_1, \dots, a_n) : a_i \geq 0 \quad \forall i = \overline{1, n}$ and $\exists i : a_i > 0$.

Finally, assume we know $n \times n$ matrices $\mathcal{E}, \tilde{\mathcal{E}}$ consisting of the following elements:

$$E_{ij}(p) = \lambda_{ij} p_j^{\chi_{ij}}, \quad \tilde{E}_{ij}(p) = \tilde{\lambda}_{ij} p_j^{\tilde{\chi}_{ij}}, \quad i, j = \overline{1, n}$$

where $\lambda_{ij}, \tilde{\lambda}_{ij}, \chi_{ij}, \tilde{\chi}_{ij} \in \mathbb{R}$. Here E_{ij} is the j -th price elasticity of i -th good's demand, and $\tilde{E}_{ij}$ the j -th price elasticity of i -th good's supply.

Definition 1. An open market model is the following set

$$\sigma = (a, c_1, c_2, p^*, S^*, D^*, \mathcal{E}(p), \tilde{\mathcal{E}}(p)).$$

Theorem 1. The set σ uniquely defines the demand and the supply functions:

$$D_i(p) = D_i^* \prod_{j=1}^n \exp \left(\frac{\lambda_{ij}}{\chi_{ij}} (p_j^{\chi_{ij}} - p_j^{*\chi_{ij}}) \right), \quad i = \overline{1, n}; \quad (3)$$

$$S_i(p) = S_i^* \prod_{j=1}^n \exp \left(\frac{\tilde{\lambda}_{ij}}{\tilde{\chi}_{ij}} (p_j^{\tilde{\chi}_{ij}} - p_j^{*\tilde{\chi}_{ij}}) \right), \quad i = \overline{1, n}. \quad (4)$$

The mappings D and S defined by (3) and (4) are solutions to the partial differential equations systems:

$$\frac{\partial D_i}{\partial p_j}(p) = \frac{E_{ij}(p)D_i(p)}{p_j}, \quad i, j = \overline{1, n}; \quad (5)$$

and

$$\frac{\partial S_i}{\partial p_j}(p) = \frac{\tilde{E}_{ij}(p)S_i(p)}{p_j}, \quad i, j = \overline{1, n}. \quad (6)$$

with initial conditions (1) and (2), respectively.

Proof. We present the proof for the mapping D , the proof for S is similar. First, we show that (3) is a solution to the problem (1),(5). To do that, we calculate:

$$\begin{aligned} \frac{\partial D_i}{\partial p_k}(p) &= \frac{\lambda_{ik}}{p_k} D_i^* \prod_{j=1}^n \exp \left(\frac{\lambda_{ij} p_k^{\chi_{ik}}}{\chi_{ij}} (p_j^{\chi_{ij}} - p_j^{*\chi_{ij}}) \right) = \\ &= \lambda_{ik} p_k^{\chi_{ik}} \frac{D_i(p)}{p_k}, \quad i = \overline{1, n}. \end{aligned}$$

Moreover,

$$D_i(p^*) = D_i^* \prod_{j=1}^n \exp \left(\frac{\lambda_{ij}}{\chi_{ij}} (p_j^{*\chi_{ij}} - p_j^{*\chi_{ij}}) \right) = D_i^*, \quad i = \overline{1, n}.$$

Now prove that the solution to the problem (1), (5) is unique in the model σ . To do that, consider the equation:

$$\begin{aligned} d \left(D_i(p) \prod_{j=1}^n \exp \left(-\frac{\lambda_{ij}}{\chi_{ij}} p_j^{\chi_{ij}} \right) \right) = \\ \sum_{k=1}^n \left(\frac{\partial D_i}{\partial p_k}(p) \prod_{j=1}^n \exp \left(-\frac{\lambda_{ij}}{\chi_{ij}} p_j^{\chi_{ij}} \right) - \right. \\ \left. D_i(p) \frac{\lambda_{ik} p_k^{\chi_{ik}}}{p_k} \prod_{j=1}^n \exp \left(-\frac{\lambda_{ij}}{\chi_{ij}} p_j^{\chi_{ij}} \right) \right) dp_j = 0, \end{aligned}$$

where $i = \overline{1, n}$, which is equivalent to

$$D_i(p) \prod_{j=1}^n \exp \left(-\frac{\lambda_{ij}}{\chi_{ij}} p_j^{\chi_{ij}} \right) = C_i, \quad i = \overline{1, n}.$$

Taking into account the initial condition (1), one can see that

$$D_i^* \prod_{j=1}^n \exp \left(-\frac{\lambda_{ij}}{\chi_{ij}} p_j^{*\chi_{ij}} \right) = C_i, \quad i = \overline{1, n}.$$

This proves the solution to the problem (1), (5) is unique. $\square$

Definition 2. An equilibrium in the model σ is a vector $p^0 \in P$ such that

$$S(p^0) + a = D(p^0). \quad (7)$$

III. COVERING MAPPINGS AND COINCIDENCE POINTS

Let us recall some notions and results from the theory of covering mappings and coincidence points. First, we define covering mappings.

Let $(X, \rho_X), (Y, \rho_Y)$ be metric spaces with metrics ρ_X and ρ_Y correspondingly. Let a mapping $\Psi : X \rightarrow Y$ be given. By $B_X(x, r)$ denote the ball with the center at the point $x \in X$ and the radius $r > 0$. Similarly, we denote $B_Y(y, r)$. Finally, let α be an arbitrary positive number.

Definition 3 (Definition 1, [6]). A mapping $\Psi : X \rightarrow Y$ is called α -covering if

$$\Psi(B_X(x, r)) \supseteq B_Y(\Psi(x), \alpha r), \quad \forall x \in X, \quad r > 0.$$

It is obvious that if the mapping Ψ is α -covering, it is α' -covering for any $0 < \alpha' < \alpha$. We denote the supremum of all $\alpha > 0$ such that the mapping Ψ is α -covering by $\text{cov}(\Psi)$.

Let $M \subseteq X$ be a set with a nonempty interior.

Definition 4 (Definition 2, [6]). The mapping Ψ is called α -covering on the set $M \subseteq X$ if for any $x \in M$, $r > 0$ such that $B_X(x, r) \subseteq M$ the following inclusion holds true:

$$B_Y(\Psi(x), \alpha r) \subseteq \Psi(B_X(x, r)).$$

We denote the supremum of all $\alpha > 0$ such that the mapping Ψ is α -covering on the set M by $\text{cov}(\Psi|M)$.

Definition 5 (Definition 4, [6]). The mapping Ψ is called α -covering at the point $x \in \text{int } M$ if for any $\varepsilon > 0$ there exists $\delta \in (0, \varepsilon)$ such that $B_X(x, \delta) \subset M$ and

$$B_Y(\Psi(x), \alpha \delta) \subseteq \Psi(B_X(x, \delta)).$$

We denote the supremum of all $\alpha > 0$ such that the mapping Ψ is α -covering at the point $x \in X$ by $\text{cov}(\Psi|x)$.

In [6] it was shown that if X, Y are Banach spaces, $\Psi : X \rightarrow Y$ is strictly differentiable at any point of an open set $M \subset X$ and $\Psi'(x)$ is α -covering at any point $x \in M$ with the same constant $\alpha > 0$, then the mapping Ψ is α' -covering on M with any constant $\alpha' < \alpha$, i.e.

$$\text{cov}(\Psi|M) = \inf_{x \in \text{int } M} \text{cov}(\Psi|x). \quad (8)$$

In the case, when the mapping Ψ is a linear surjective operator from $\mathbb{R}^n$ to $\mathbb{R}^k$, the following estimate holds true:

Lemma 1 ([7]). Let $\Psi : \mathbb{R}^n \rightarrow \mathbb{R}^k$ be a surjective linear operator. Then

$$\text{cov}(\Psi) \geq \frac{1}{\|\Psi^*(\Psi\Psi^*)^{-1}\|}, \quad (9)$$

where Ψ^* is a conjugate operator to Ψ .

From now on, the norm of an arbitrary linear operator $Q : (X, \|\cdot\|_X) \rightarrow (Y, \|\cdot\|_Y)$ is defined by: $\|Q\| = \sup_{\|x\|_X=1} \|Qx\|_Y$.

Definition 6 (see [6]). A point $\xi \in X$ is a coincidence point of the mapping Ψ and some mapping $\Phi : X \rightarrow Y$ if

$$\Psi(\xi) = \Phi(\xi).$$

Now we formulate the conditions which guarantee the existence of a coincidence point for α -covering mapping and the mapping satisfying the Lipschitz condition. The following theorem is proved in [6].

Theorem 2 (Theorem 1, [6]). *Let the space X be complete, $\alpha > 0$, $x_0 \in X$, and $R > 0$. Let also $\Psi : X \rightarrow Y$ be α -covering on $B_X(x_0, R)$, and $\Psi(X)$ has a complete image. Then, for any nonnegative $\beta < \alpha$ and any mapping $\Phi : B_X(x_0, R) \rightarrow Y$ satisfying the Lipschitz condition with the Lipschitz constant β such that*

$$\rho_Y(\Psi(x_0), \Phi(x_0)) \leq (\alpha - \beta)R,$$

for mappings Ψ and Φ there exists a coincidence point $\xi \in B_X(x_0, R)$, i.e., $\Psi(\xi) = \Phi(\xi)$, and such that

$$\rho_X(x_0, \xi) \leq \frac{\rho_Y(\Psi(x_0), \Phi(x_0))}{\alpha - \beta}.$$

IV. THE MAIN RESULTS

We use the coincidence points theorem. Denote

$$\bar{\alpha}(\sigma) = \min_{p \in P} \left(\left\| \left(\frac{\partial S}{\partial p}(p) \right)^* \left(\left(\frac{\partial S}{\partial p}(p) \right) \left(\frac{\partial S}{\partial p}(p) \right)^* \right)^{-1} (p) \right\| \right)^{-1}.$$

$$\begin{aligned} \bar{\beta}(\sigma) = \max_{i=1,n} D_i^* \prod_{j=1}^n \max_{m=1,2} \left(\exp \left(\frac{\lambda_{ij}}{\chi_{ij}} (c_{mj}^{x_{ij}} - p_j^{*x_{ij}}) \right) \right) \times \\ \times \sum_{k=1}^n \frac{c_{2k} - c_{1k}}{2} \frac{|\lambda_{ik}|}{c_{1k}}, \end{aligned}$$

$$\bar{\gamma}(\sigma) = \max_{i=1,n} |S_i(\tilde{c}) + a_i - D_i(\tilde{c})|.$$

Theorem 3. *Let the parameters of the model σ satisfy the following conditions:*

- 1) $\det \frac{\partial S}{\partial p}(p) \neq 0$;
- 2) $\bar{\gamma}(\sigma) < \bar{\alpha}(\sigma) - \bar{\beta}(\sigma)$.

Then there exists an equilibrium $p^0 \in \text{int } P$ in the model σ .

Proof. In the space $\mathbb{R}_+^n$ define two norms:

$$\|x\|_1 = \max_{i=1,n} \frac{2|x_i|}{c_{2i} - c_{1i}}, \quad x = (x_1, \dots, x_n), \quad (10)$$

$$\|y\|_2 = \max_{i=1,n} |y_i|, \quad y = (y_1, \dots, y_n). \quad (11)$$

Consider the metric spaces $(X, \rho_X), (Y, \rho_Y)$ where $X = Y = \mathbb{R}_+^n$, and the metrics ρ_X and ρ_Y are generated by the norms $\|\cdot\|_1$ and $\|\cdot\|_2$ by (10) and (11), respectively. Our goal is to apply Theorem 2. To do that, we want to estimate $\text{cov}(S|P)$. By the results from [6] we have

$$\text{cov}(S|P) = \min_{p \in P} \text{cov}(S|p)$$

and

$$\text{cov}(S|p) = \text{cov} \left(\frac{\partial S}{\partial p}(p) \right).$$

In turn, by Lemma 1 we have

$$\begin{aligned} \text{cov} \left(\frac{\partial S}{\partial p}(p) \right) &\geq \\ &\geq \left(\left\| \left(\frac{\partial S}{\partial p}(p) \right)^* \left(\left(\frac{\partial S}{\partial p}(p) \right) \left(\frac{\partial S}{\partial p}(p) \right)^* \right)^{-1} (p) \right\| \right)^{-1} = \bar{\alpha}(p). \end{aligned}$$

Therefore,

$$\text{cov}(S|P) = \min_{p \in P} \text{cov}(S|p) \geq \min_{p \in P} \bar{\alpha}(p) = \bar{\alpha}(\sigma).$$

Now we estimate $\text{lip}(D|P)$. First we calculate

$$\begin{aligned} \left\| \frac{\partial D}{\partial p}(p) \right\| &= \max_{\|x\|_1=1} \left\| \frac{\partial D}{\partial p}(p)x \right\| = \\ &= \max_{\|x\|_1=1} \max_{i=1,n} \left| \sum_{j=1}^n \frac{\partial D_i}{\partial p_j}(p)x_j \right| = \\ &= \max_{\|x\|_1=1} \max_{i=1,n} \left| \sum_{j=1}^n \lambda_{ij} p_j^{x_{ij}} \frac{D_i}{p_j} x_j \right| = \\ &= \max_{\|x\|_1=1} \max_{i=1,n} \left| \sum_{j=1}^n \lambda_{ij} p_j^{x_{ij}-1} x_j D_i^* \times \right. \\ &\quad \left. \prod_{k=1}^n \exp \left(\frac{\lambda_{ik}}{\chi_{ik}} (p_k^{x_{ik}} - (p_k^*)^{x_{ik}}) \right) \right| \leq \\ &= \max_{i=1,n} \sum_{j=1}^n |\lambda_{ij}| \max \left\{ c_{1j}^{x_{ij}-1}, c_{2j}^{x_{ij}-1} \right\} D_i^* \times \\ &\quad \prod_{k=1}^n \max_{m=1,2} \exp \left(\frac{\lambda_{ik}}{\chi_{ik}} (c_{mk}^{x_{ik}} - (p_k^*)^{x_{ik}}) \right) = \bar{\beta}(\sigma). \end{aligned}$$

Hence,

$$\text{lip}(D|P) = \max_{p \in P} \frac{\partial D}{\partial p}(p) \leq \bar{\beta}(\sigma).$$

From 2) and the inequalities

$$\text{cov}(D) \geq \bar{\alpha}(\sigma), \quad \text{lip}(S) \leq \bar{\beta}(\sigma)$$

we conclude that there exist α, β such that $\bar{\beta}(\sigma) < \beta < \alpha < \bar{\alpha}(\sigma)$, the mapping S is α -covering on P , and the mapping β satisfies the Lipschitz condition on P with the constant β . Since the space (P, ρ_X) is complete, by Theorem 2 there exists a vector $p^0 \in \text{int } P$ such that

$$S(p^0) + a = D(p^0),$$

and

$$\rho_X(p^0, \tilde{c}) \leq \frac{\rho_Y(D(\tilde{c}), S(\tilde{c}))}{\alpha - \beta} < 1$$

by 2). Hence, in the model σ there exists an equilibrium $p^0 \in \text{int } P$. $\square$

V. THE SPECIAL CASE. LINEAR ELASTICITIES

Let

$$\chi_{ij} = \tilde{\chi}_{ij} = 1, \quad i, j = \overline{1, n}. \quad (12)$$

In this case we also may obtain the uniqueness of equilibrium. Denote $\Lambda = (\tilde{\lambda}_{ij})_{i,j=\overline{1,n}}$ and implement an operator $B : \mathbb{R}_+^n \rightarrow \mathbb{R}_+^n$ by:

$$B = S^{-1} \circ (D - a). \quad (13)$$

Theorem 4. *Let the parameters of the model σ satisfy the following conditions:*

- 1) $\det \Lambda \neq 0$;
- 2) $B(P) \subset P$;
- 3) $\text{lip}(B|P) < 1$.

Then in the model σ there exists a unique equilibrium $p^0 \in P$.

Proof. Show that S is bijective. To do that, we consider the following system of equations:

$$S_i^* \prod_{j=1}^n \exp(\tilde{\lambda}_{ij}(p_j - p_j^*)) = s_i, \quad i = \overline{1, n}. \quad (14)$$

By taking logarithm of the both sides, we get

$$\ln S_i^* + \sum_{j=1}^n \tilde{\lambda}_{ij}(p_j - p_j^*) = \ln s_i, \quad i = \overline{1, n}. \quad (15)$$

Therefore, we obtain the system of linear equations:

$$\sum_{j=1}^n \tilde{\lambda}_{ij} p_j = \ln \frac{s_i}{S_i^*} + \sum_{j=1}^n \tilde{\lambda}_{ij} p_j^*, \quad i = \overline{1, n}. \quad (16)$$

Since the matrix Λ is nondegenerate, system (16) is compatible and it has a unique solution:

$$p_i = \sum_{j=1}^n \tilde{\mu}_{ij} \ln \frac{s_j}{S_j^*} + p_i^*, \quad j = \overline{1, n} \quad (17)$$

where μ_{ij} is the element of the inverse matrix to Λ .

Since the mapping S is bijective, there exists the inverse mapping S^{-1} . Therefore, the mapping B defined by (13) is also exists. Moreover, by 2) and 3) the mapping $S^{-1} \circ (D - a)$ is a contraction. Therefore, by the Banach theorem there exists a unique point $p^0 \in P$ such that (7) holds true. $\square$

Remark 1. The calculation of the Lipschitz mapping B may be difficult. We obtain an estimation for this constant.

Denote:

$$\begin{aligned} \bar{\alpha}(\sigma_{of}^1) &= \left(\max_{i=\overline{1,n}} \frac{2}{c_{2i} - c_{1i}} \sum_{k=1}^n \frac{|\tilde{\mu}_{ik}|}{S_k^*} \times \right. \\ &\times \left. \prod_{j=1}^n \max_{m=1,2} \left\{ \exp(\tilde{\lambda}_{kj}(p_j^* - c_{mj})) \right\} \right)^{-1}; \end{aligned}$$

$$\begin{aligned} \bar{\beta}(\sigma_{of}^1) &= \max_{i=\overline{1,n}} D_i^* \prod_{j=1}^n \max_{m=1,2} \left\{ \exp(\lambda_{ij}(c_{mj} - p_j^*)) \right\} \times \\ &\times \sum_{k=1}^n \frac{2|\lambda_{ik}|}{c_{2k} - c_{1k}}; \\ \bar{\gamma}(\sigma_{of}^1) &= \max_{i=\overline{1,n}} |S_i(\tilde{c}) + a_i - D_i(\tilde{c})|. \end{aligned}$$

Theorem 5. *Let the parameters of the model σ satisfy the following conditions:*

- 1) $\det \Lambda \neq 0$;
- 2) $\bar{\gamma}(\sigma) < \bar{\alpha}(\sigma) - \bar{\beta}(\sigma)$.

Then in the model σ there exists a unique equilibrium $p^0 \in P$.

Proof. First we estimate $\text{lip}(S^{-1}|S(P))$. From (17) we have

$$S_i^{-1}(s) = p_i^* + \sum_{j=1}^n \tilde{\mu}_{ij} \ln \frac{s_j}{S_j^*}.$$

Then for any $s \in S(P)$

$$\frac{\partial S_i^{-1}}{\partial s_k}(s) = \frac{\tilde{\mu}_{ik}}{s_k}.$$

Hence,

$$\begin{aligned} \left\| \frac{\partial S^{-1}}{\partial s}(s) \right\| &= \max_{\|x\|_2=1} \left\| \frac{\partial S}{\partial s}(s) \right\| = \\ &= \max_{\|x\|_2=1} \max_{i=\overline{1,n}} \frac{2}{c_{2i} - c_{1i}} \left| \sum_{k=1}^n \frac{\partial S_i}{\partial s_k}(s) x_k \right| \leq \\ &= \max_{\|x\|_2=1} \max_{i=\overline{1,n}} \frac{2}{c_{2i} - c_{1i}} \sum_{k=1}^n |\mu_{ik}| \frac{|x_k|}{s_k} \leq \\ &\leq \max_{i=\overline{1,n}} \frac{2}{c_{2i} - c_{1i}} \sum_{k=1}^n \frac{|\mu_{ik}|}{c_{1k}} = \bar{\beta}(\sigma). \end{aligned}$$

Therefore,

$$\text{lip}(S^{-1}|S(P)) = \max_{s \in S(P)} \left\| \frac{\partial S^{-1}}{\partial s}(s) \right\| \leq \frac{1}{\bar{\alpha}(\sigma)}.$$

Now we estimate $\text{lip}(D|P)$. First, we have

$$\begin{aligned} \left\| \frac{\partial D}{\partial p}(p) \right\| &= \max_{\|x\|_1=1} \left\| \frac{\partial D}{\partial p}(p) x \right\|_2 = \\ &= \max_{\|x\|_1=1} \max_{i=\overline{1,n}} \left| \sum_{k=1}^n \frac{\partial D_i(p)}{\partial p_k} x_k \right| = \\ &= \max_{\|x\|_1=1} \max_{i=\overline{1,n}} \left| \sum_{k=1}^n \lambda_{ik} D_i(p) x_k \right| \leq \\ &\leq \max_{i=\overline{1,n}} \left| \sum_{k=1}^n \lambda_{ik} x_k D_i^* \prod_{j=1}^n \exp(\lambda_{ij}(p_j - p_j^*)) \right| \leq \\ &\leq \max_{i=\overline{1,n}} D_i^* \prod_{j=1}^n \max_{m=1,2} \left\{ e^{\lambda_{ij}(c_{mj} - p_j^*)} \right\} \sum_{k=1}^n \frac{c_{2k} - c_{1k}}{2} |\lambda_{ik}| \\ &= \bar{\beta}(\sigma). \end{aligned}$$

Hence,

$$\text{lip}(D|P) = \max_{p \in P} \left\| \frac{\partial D}{\partial p}(p) \right\| \leq \bar{\beta}(\sigma).$$

From 2) and the inequalities

$$\text{lip}(S^{-1}|S(P)) \leq \frac{1}{\bar{\alpha}(\sigma)}, \quad \text{lip}(D|P) \leq \bar{\beta}(\sigma)$$

we get that there exist $\alpha > 0, \beta > 0$ such that $\bar{\gamma}(\sigma) < \alpha - \beta$, the mapping S^{-1} satisfies the Lipschitz condition on $S(P)$ with the constant $1/\alpha$, and the mapping D satisfies the Lipschitz condition on P with the constant $\bar{\beta}(\sigma)$.

By repeating the proof of Theorem 2 we obtain that

$$\text{lip}(B|P) \leq \text{lip}(S^{-1}|S(P)) \cdot \text{lip}(D|P).$$

and $B(P) \subseteq P$. Since

$$\text{lip}(B|P) \leq \frac{1}{\alpha} \beta < 1$$

by 2) we obtain that the mapping B is a contraction. Since (P, ρ_X) is a complete metric space, by the Banach theorem there exists a unique solution to the equation $B(p) = p$. Therefore, in the model σ_{of}^1 there exists a unique equilibrium p^0 . $\square$

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